Introduction of the Bezgin Method to estimate dynamic impact forces due to track and wheel roughness

Niyazi Özgür Bezgin, Ph.D

ISTANBUL UNIVERSITY - CERRAHPAŞA

Associate Professor of Civil Engineering

ozgur.bezgin@istanbul.edu.tr

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Dr. Niyazi <u>Özgür</u> Bezgin

Associate Professor in Civil Engineering
Istanbul University – Cerrahpaşa

Concentration on:
1. *Design of prefabricated structures*
2. *Soil and structure interaction*
3. *Railway track mechanics*

ozgur.bezgin@istanbul.edu.tr

*An avid fan of the sea
and things that relate
to the sea.*

*Long distance
runner. Running half-
marathons since
2007.*

CONTENT

1. **Motivation** to develop a simple analytical tool to estimate dynamic impact forces on railway tracks due to track and wheel roughness.

2. **Bezgin Method** and its physical foundations.

3. **Extended Bezgin Equations** that estimate dynamic impact forces due to track and wheel roughness.

4. **ALLTRACK v1**

ADVANCED METHODS TO ESTIMATE VERTICAL FORCES

- **Multi-body simulation softwares** – They are necessary…Complicated... Contact mechanics...Deeper scientific extent... Estimates with higher precision… Useful for design finalization and maintenance assesment.

  - **Simpack®**
  - **Vampire®**
  - **Universal Mechanism®**
  - **Simulia®**
  - **ANSYS®**
  - **ABAQUS®**

- **However…** They are so capable that one may unconditionally yield to their estimates thereby relinquishing the engineering judgement…

- The programs may be not be available.

ADVANCED METHODS TO ESTIMATE VERTICAL FORCES

- **Time** required to construct a valid model.

- Time required for analysis.

- **Cost** of sophisticated software and the powerful hardware.

- **Availability** of the programs (budget).

- Availability of the **qualified** personnel to develop the models and operate the programs.

SOME EMPIRICAL METHODS TO ESTIMATE VERTICAL FORCES

$K = 1 + 0.33\frac{v}{D}$ (Imperial)

$K = 1 + 0.33\frac{v}{D} * \frac{1}{1.609} * 25.4 \rightarrow K = 1 + 5.21\frac{v}{D}$ (SI)   *Talbot*

Eisenmann $K = 1 + \varphi.n.t$

$\varphi = 1 + 0.5\frac{V-60}{190} \left(60 < v < 300\frac{km}{h}\right)$ and $\varphi = 1$ $(v \leq 60 \text{ km/h})$

$K = 1 + \frac{15v}{D\sqrt{U}}$ (Imperial)

$K = 1 + \frac{15v}{D\sqrt{U}} * \frac{1}{1.609} * 25.4 * 0.08303 \rightarrow K = 1 + \frac{19.7v}{D\sqrt{U}}$ (SI)   *Clarke*

Deutsche Bahn $K = 1 + \frac{11.655v^2}{10^5} - \frac{6.252v^3}{10^7}$

$K = 1 + n.t\frac{v}{200}$, for n=0.3 and t=3, $K = 1 + 0.0045v$ *Japanese Railways*

EMPIRICAL ESTIMATES OF VERTICAL FORCES

- **Empirical equations** – Simple… implicit characteristics…they rarely come with users manuals…variable estimates
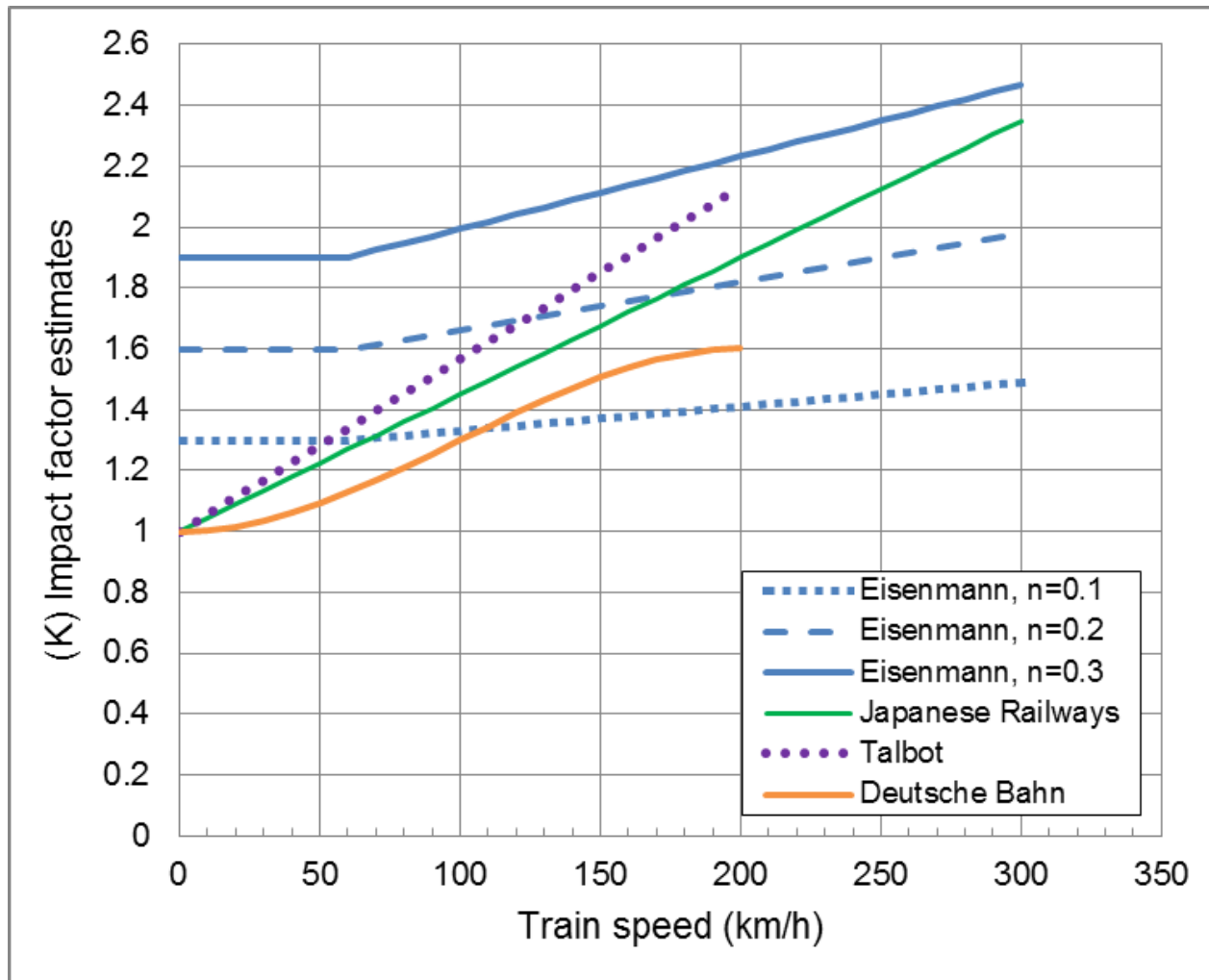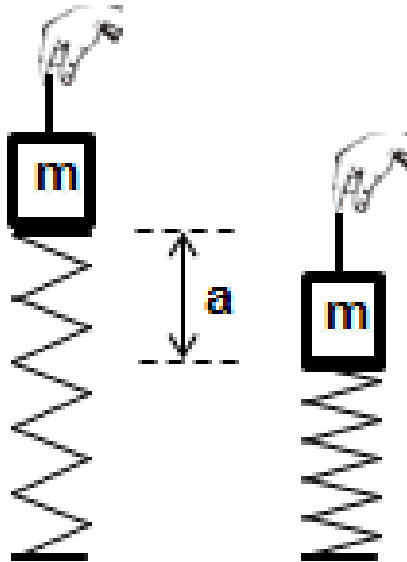
$$W = m.\,g$$

$$F_s = k.\,a$$

$$a = \frac{m.\,g}{k}$$

- Potential energy of the mass (P_m) releases into the spring to be stored as potential energy of the deformed spring (P_s).
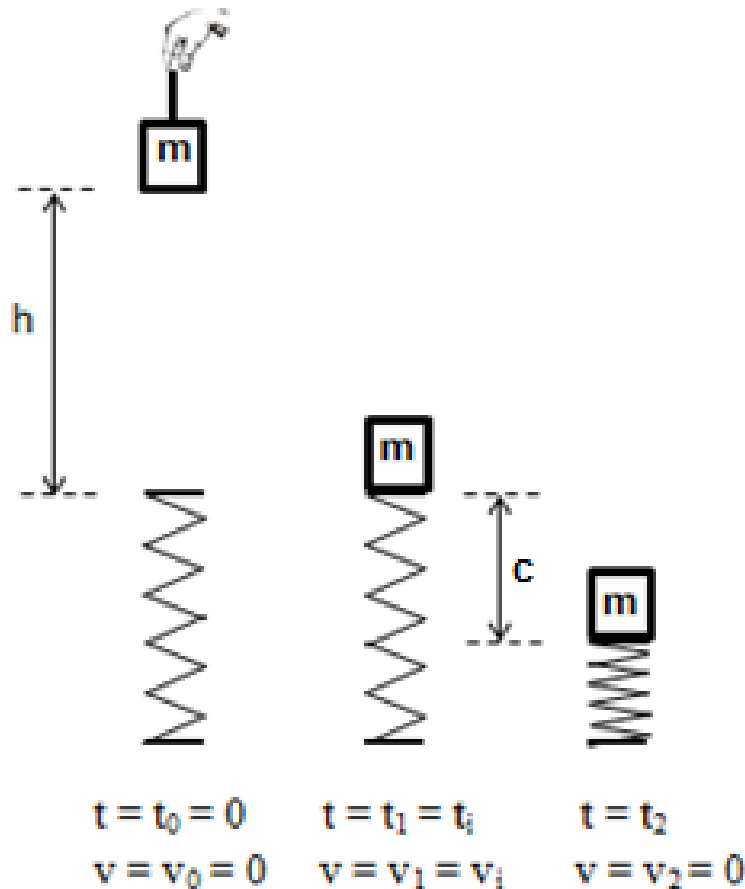
$$P_m = m.g.h + \int_0^c m.g.\,dx$$

$$P_s = \int_0^c k.x.\,dx = \frac{1}{2}k.c^2$$

$t = t_0 = 0$ $\quad$ $t = t_1 = t_i$ $\quad\quad$ $t = t_2$ $\quad\quad\quad\quad$ $t = t_3$

$v = v_0 = 0$ $\quad$ $v = v_1 = v_i$ $\quad$ $v = v_2 = 0$ $\quad\quad$ $v = v_3 = 0$
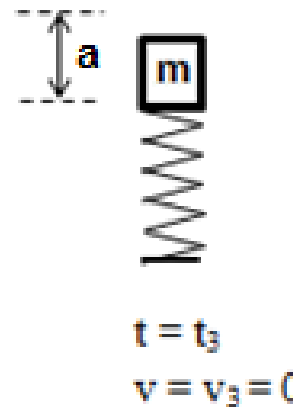
DYNAMIC IMPACT DISPLACEMENT AND DYNAMIC IMPACT FORCE

- For a given free-fall height of **'h'**, the impact displacement and the impact force are directly related to the supporting structure's stiffness **'k'**.

$$F_i = F_s \cdot \left(1 + \sqrt{2h/a + 1} \right) \text{ where } a = \frac{m \cdot g}{k}$$

1. Impact force is directly proportional to stiffness.

2. **Impact force is directly proportional to h/a.**

- <u>h</u> has a very important implication in terms of track imperfection.

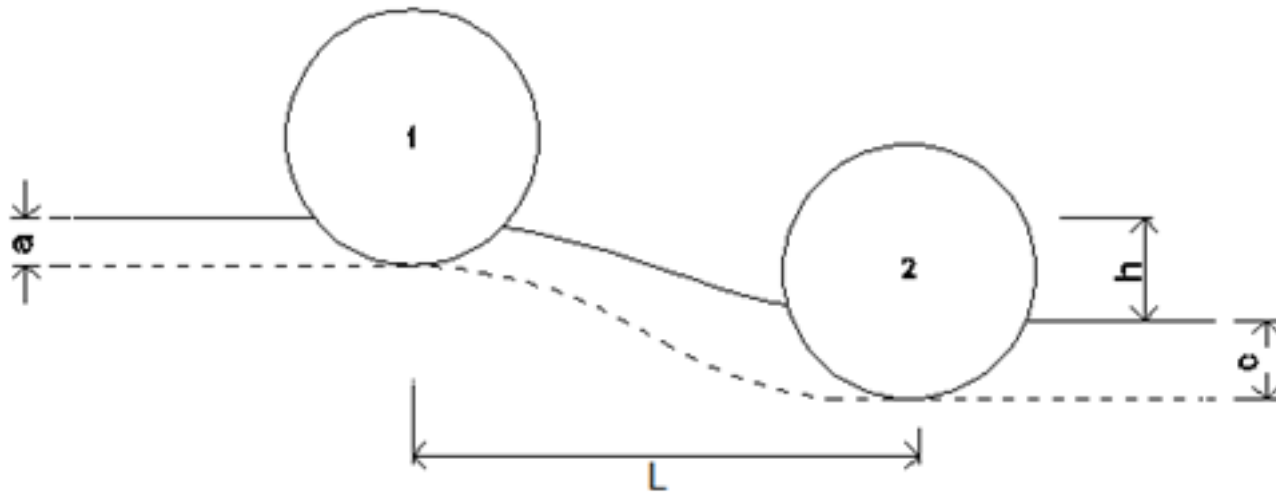*With kind rememberance of the contributions of the late **Professor. Egor Popov***

TRACK VERTICAL STIFFNESS

- Simply stating, the track stiffness per wheel is the ratio of the force applied by the wheel to the vertical rail displacement under the wheel.
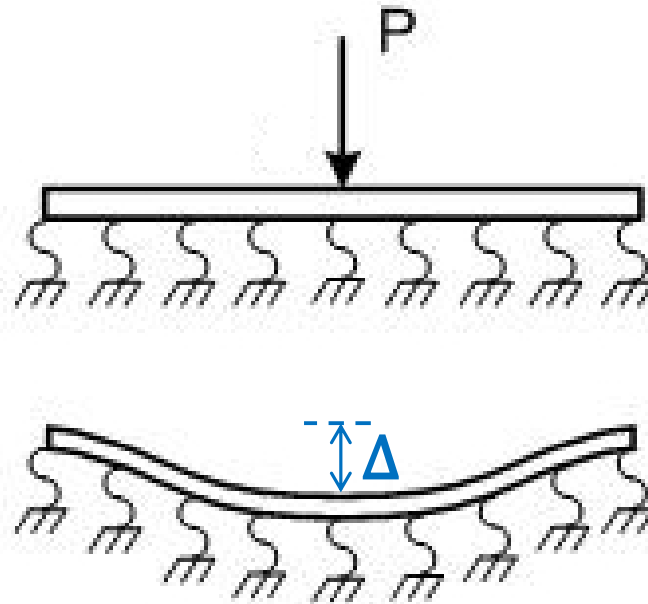
$$k = \frac{P}{\Delta}$$

$$f = 1 - \frac{t_{fall}}{t_{pass}} = 1 - \frac{v}{L}\sqrt{\frac{2h}{g}}$$

$$K_B = 1 + \sqrt{\frac{2h}{a}(1-f)}$$

DEFINITION OF «f» THROUGH KINEMATICS
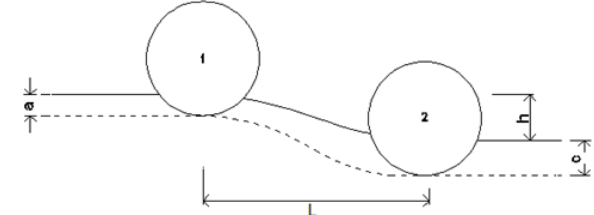
$$t_{fall} = \sqrt{\frac{2h}{g}}$$

$$f = 1 - \frac{t_{fall}}{t_{pass}}$$

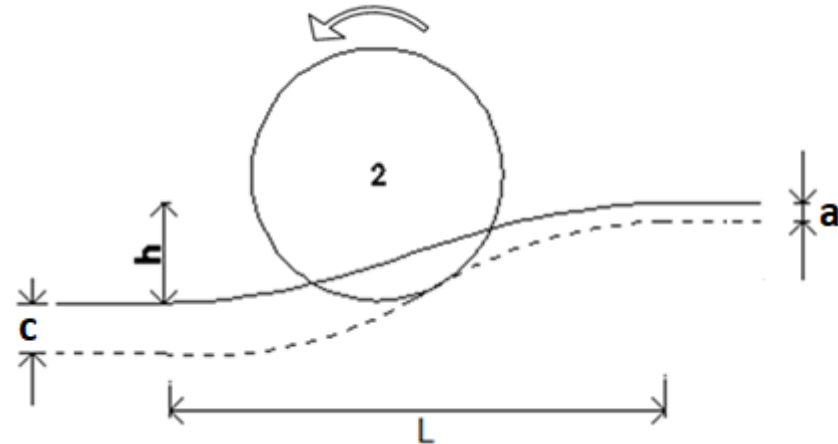$$f = 1 - \frac{v}{L}\sqrt{\frac{2h}{g}}$$

$$t_{pass} = \frac{L}{v}$$

- **Important findings:**

  1. For a given speed (v), the smaller the length (L), the smaller is the f

  2. For a given (v) and (L), the higher the (h), the smaller is the f.

TRANSFERRED PART OF POTENTIAL ENERGY TO THE TRACK

- Part of the potential energy of the tributary mass of moving train axle **releases into the track** to be stored as potential energy of the deformed track.

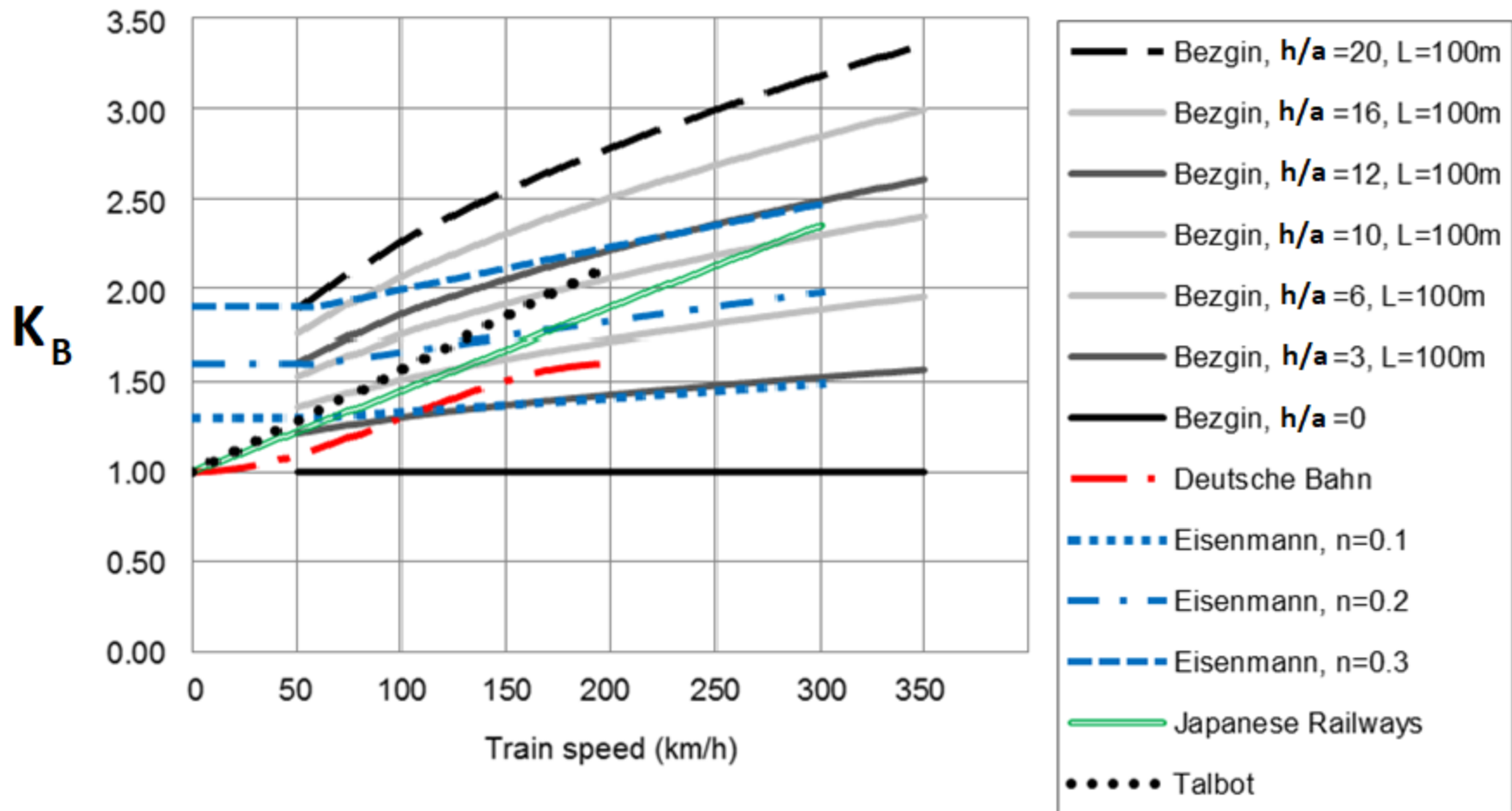$$m.\, g.\, (h + c - a) - m.\, g.\, f.\, h = \frac{1}{2} k.\, (a + c)(c - a)$$

DYNAMIC IMPACT DISPLACEMENT DUE TO TRANSFERRED POTENTIAL ENERGY AND DYNAMIC IMPACT FORCE FACTOR K_B

$$c = a\left(1 + \sqrt{\frac{2h}{a}(1-f)}\right)$$
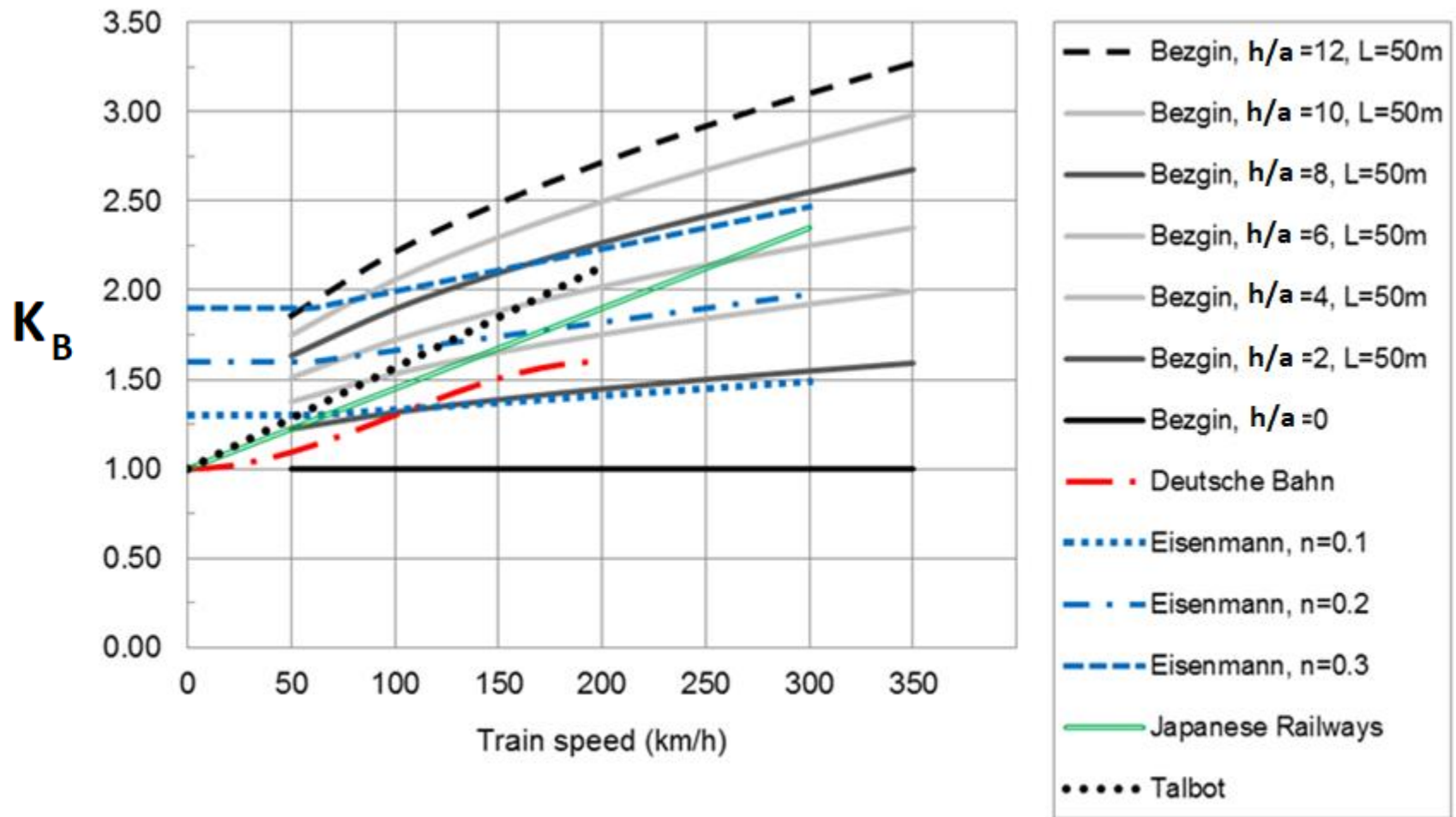
$$K_B = \left(1 + \sqrt{\frac{2h}{a}(1-f)}\right)$$

DEVELOPMENT OF THE BEZGIN METHOD AFTER MAY 2017

1. The proposed method was initially applied to a **descending track profile condition**, considering only **the stiffness of the railway track**.

2. The method was then extended to include the effects of **ascending track profile**, **increasing track stiffness**, **decreasing track stiffness** and **wheel flats** on the dynamic impact forces.

3. The method was further extended to include the effects of **stiffness values of the rolling stock suspension elements** and the **Hertzian contact stiffness**.

4. This extension of the method application yielded the **Extended Bezgin Equations** presented in the **98th TRB** on January 2019 and published in **TRR** afterwards.

GOAL: Develop a simple analytical tool to estimate the dynamic impact forces due to stiffness transitions and profile variations along railway tracks and wheel flats
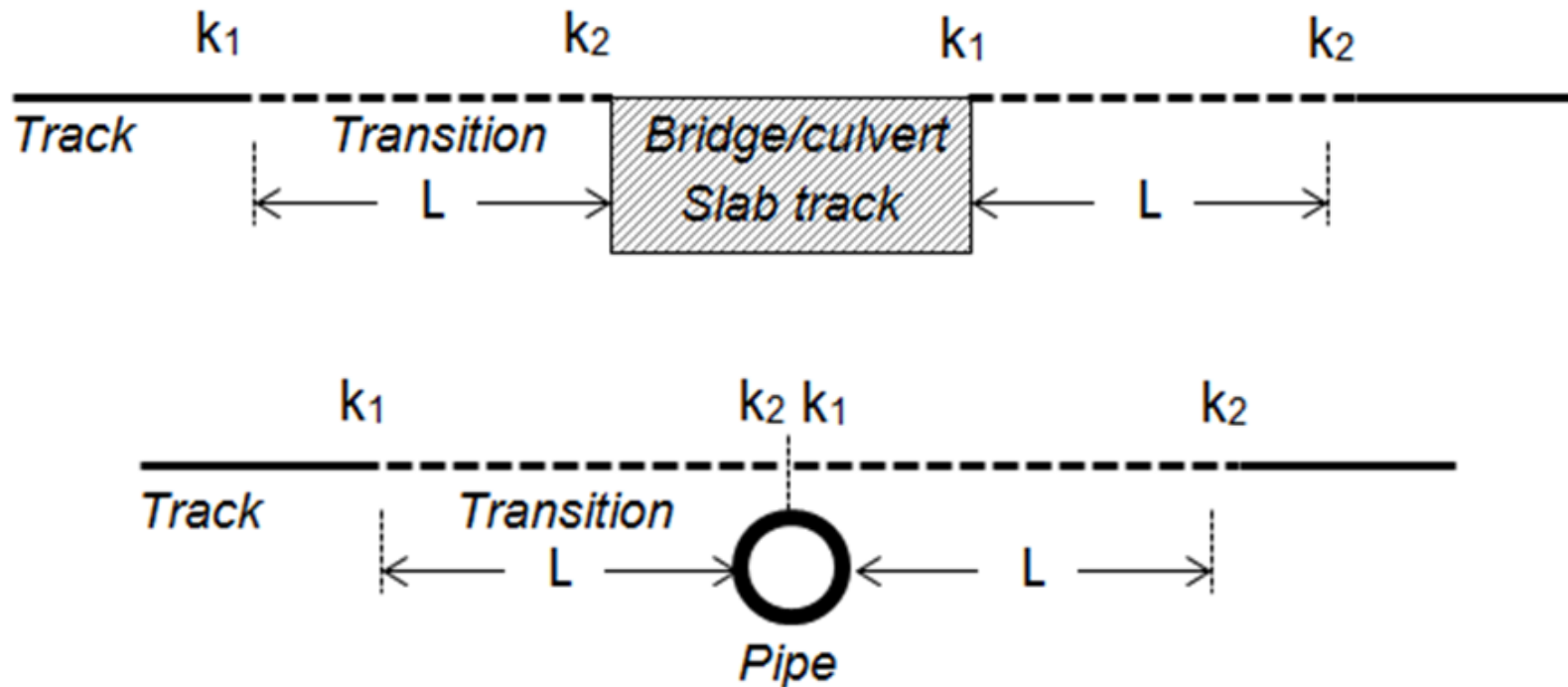
- An **ascending profile** frequently accompanies an **increasing stiffness** transition and **a descending profile** accompanies a **decreasing stiffness** transition.

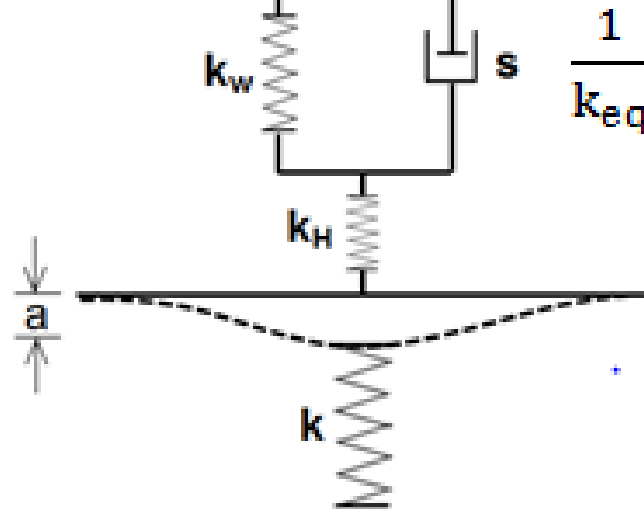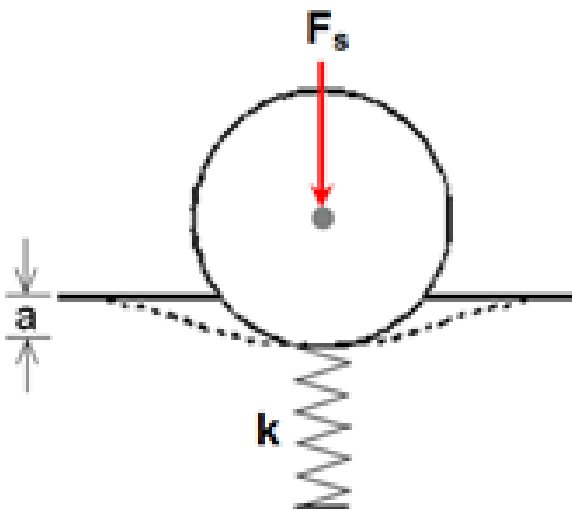# REPRESENTING THE SYSTEM STIFFNESS AND SYSTEM DAMPING

- The additional springs that form the **track-rolling stock system** provide additional locations where released potential energy of the **tributary wheel mass** can store.

- The combined stiffness of the system is k_{eq}.
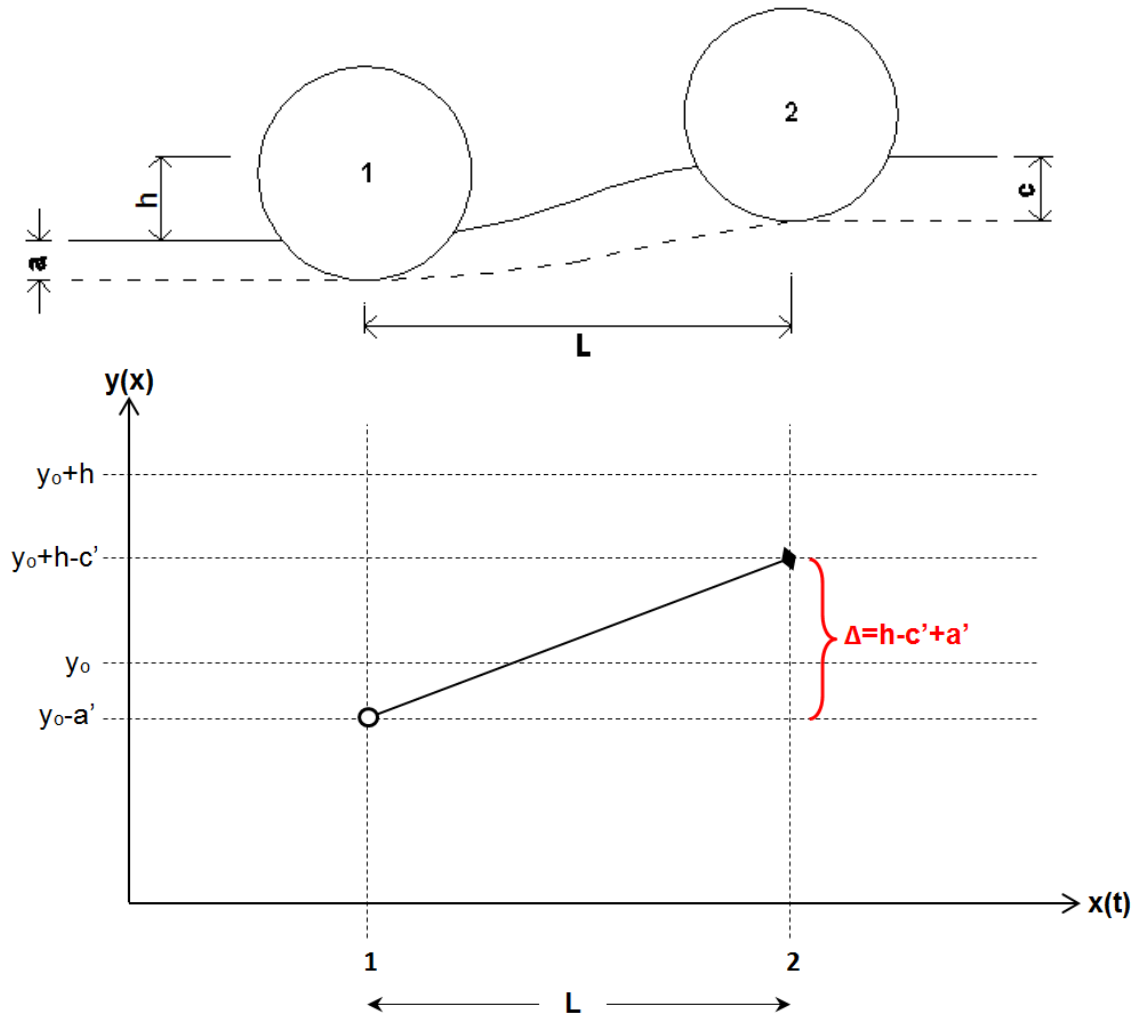
- Damping percentage parameter **s≤5%**

The combined deflection of system is:
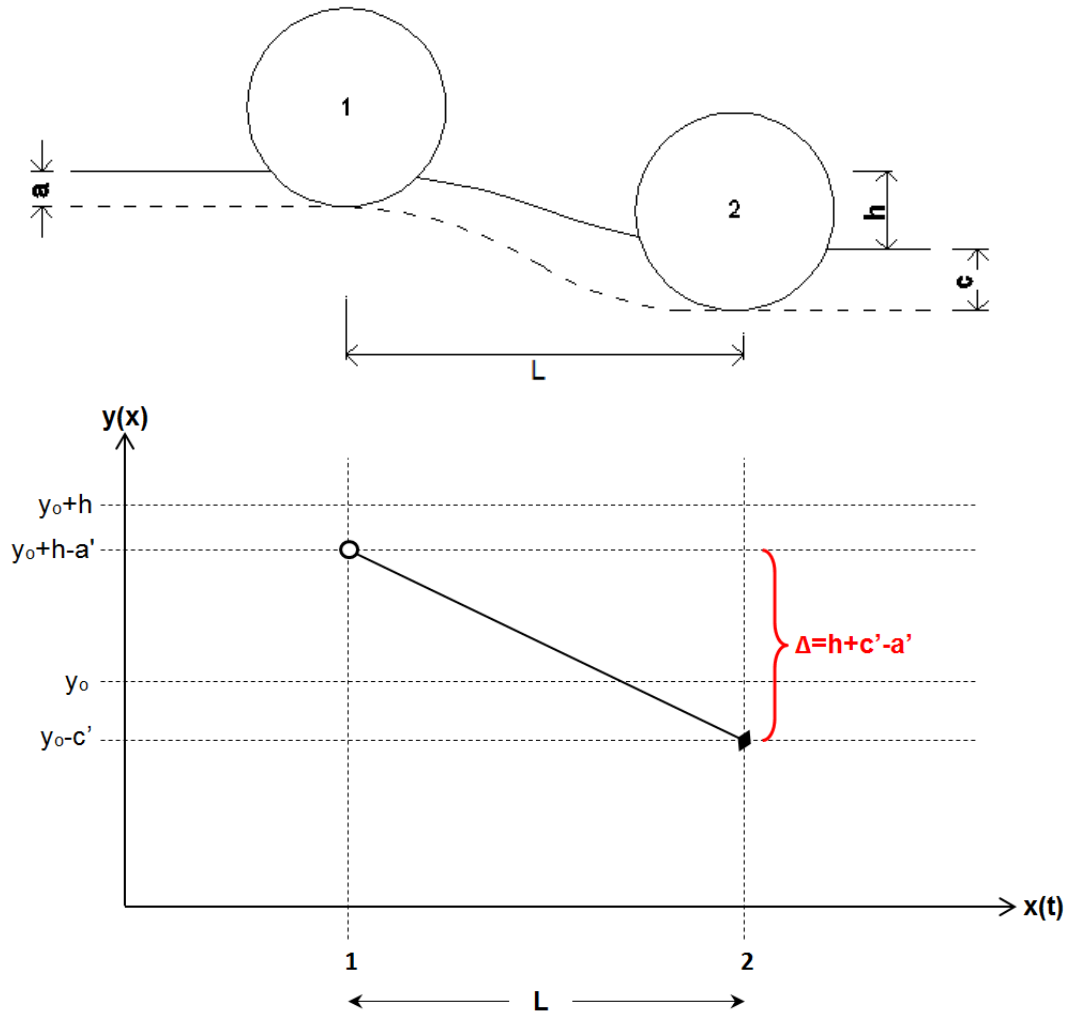
$$a' = a + a_H + a_w + a_b = F_s / k_{eq}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_w} + \frac{1}{k_H} + \frac{1}{k}$$

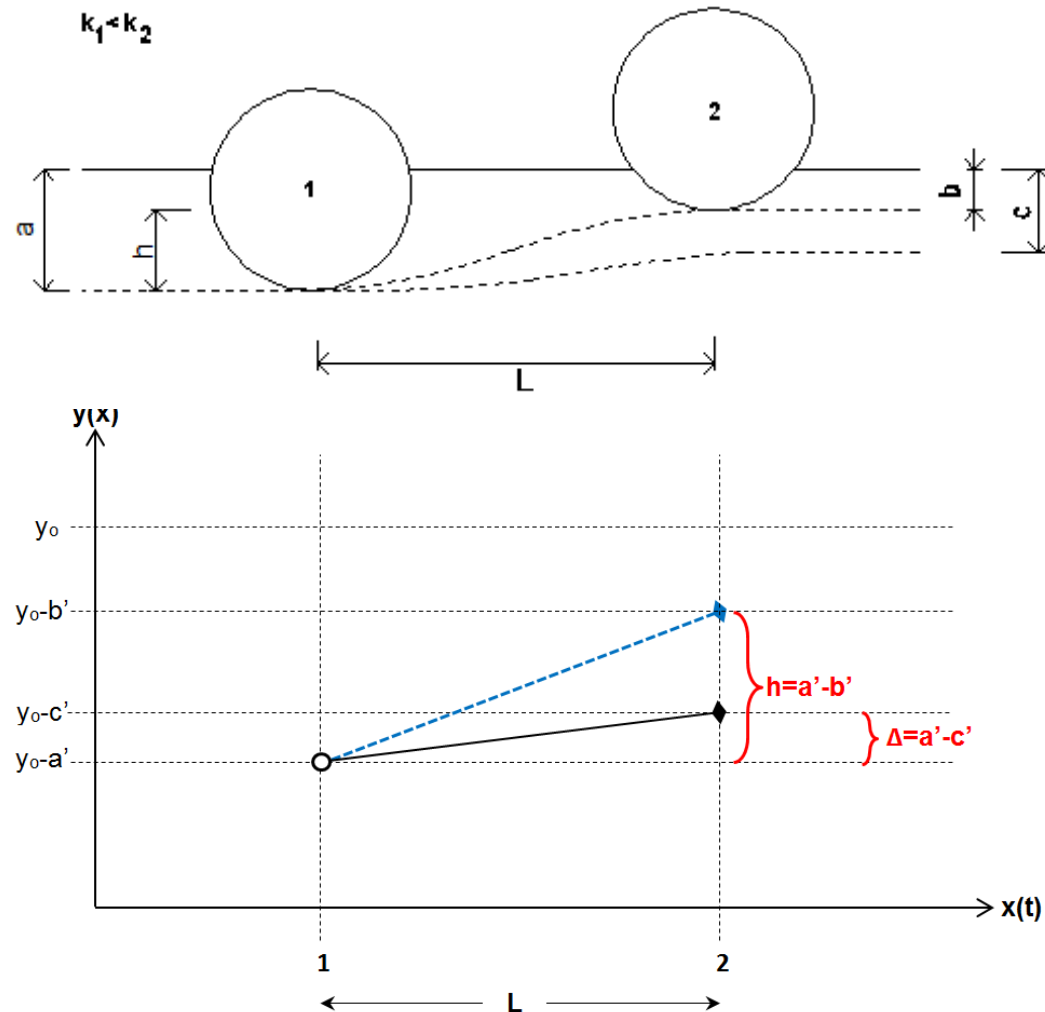# ASCENDING TRACK PROFILE AND GRAPHICAL PRESENTATION OF P.E VARIATION

$$\mathbf{m.\,g.\,(h - c' + a') - m.\,g.\,h.\,f - m.\,g.\,h.\,s = \frac{1}{2}k_{eq}(a' + c')(c' - a')}$$

$$\mathbf{m.g.(h + c' - a') - m.g.h.f - m.g.h.s = \frac{1}{2}k_{eq}(a' + c')(c' - a')}$$

$$\mathbf{m.g.\,(a'-c')-m.g.\,(a'-b').f-m.g.\,(a'-b').s=\frac{(k_{eq2}.b'+k_{eq2}.c')}{2}.(c'-b')}$$
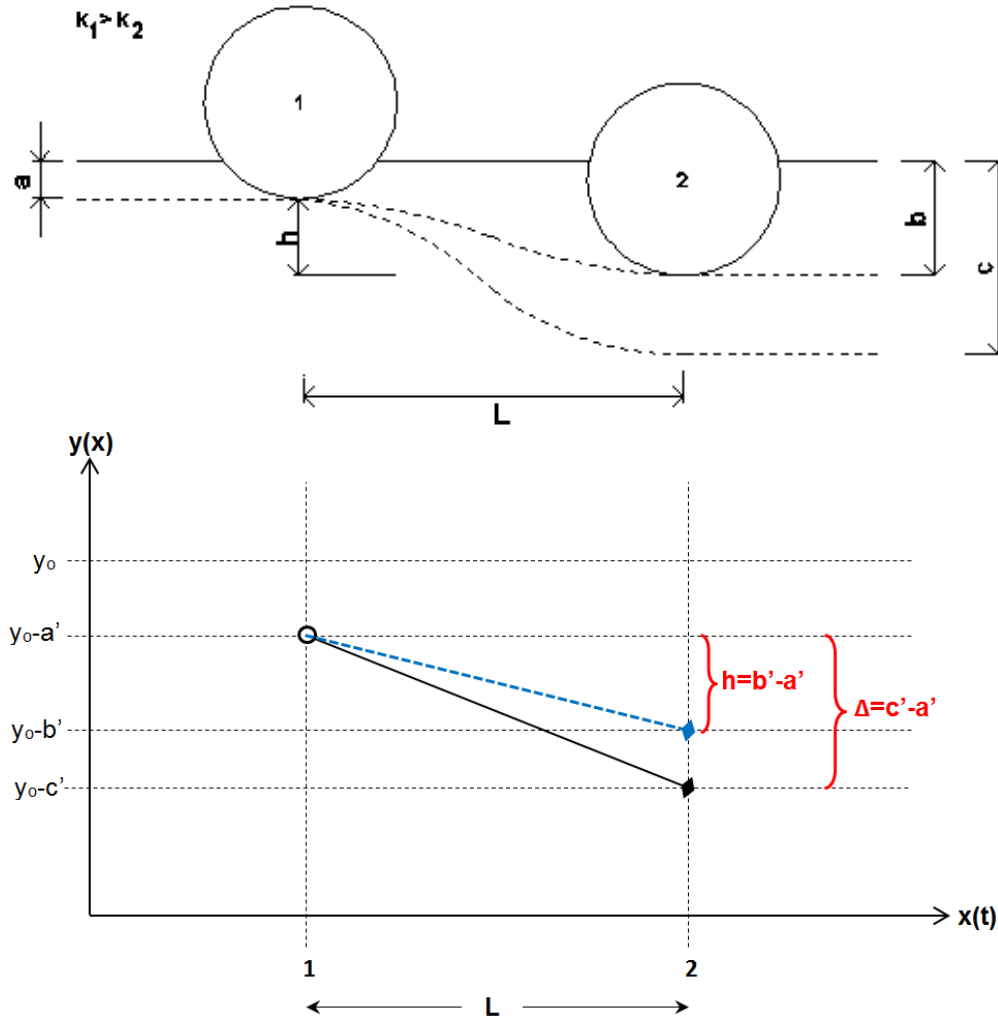
$$\mathbf{m. g.\,(c' - a') - m. g.\,(b' - a').\,f - m. g.\,(b' - a').\,s = \frac{(k_{eq2}.b' + k_{eq2}.c')}{2}.\,(c' - b')}$$

EXTENDED BEZGIN EQUATIONS FOR VARYING TRACK PROFILE AND STIFFNESS
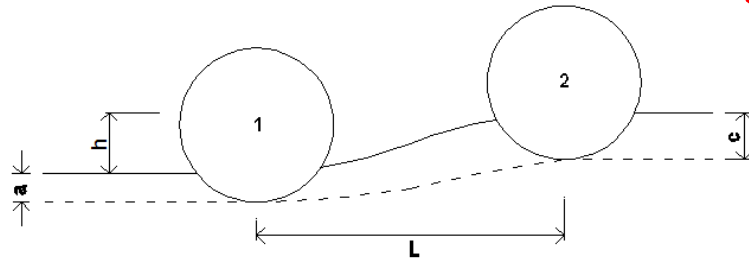
$$K'_{B,a} = 2\sqrt{\frac{h}{2a'}(1-f-s)+1} - 1$$
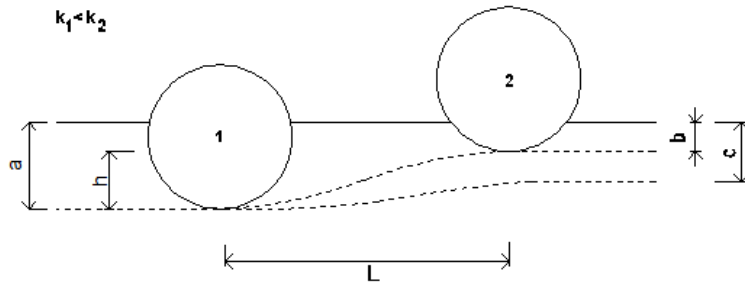
$$K'_{B,d} = 1 + \sqrt{\frac{2h}{a'}(1-f-s)}$$

$$K'_{B2} = \sqrt{2\left[1+f+s+\frac{a'}{b'}\cdot(1-f-s)\right]-1}$$

$$K'_{B1} = 1 + \sqrt{2\left[(1-f-s)\left(1-\frac{a'}{b'}\right)\right]}$$

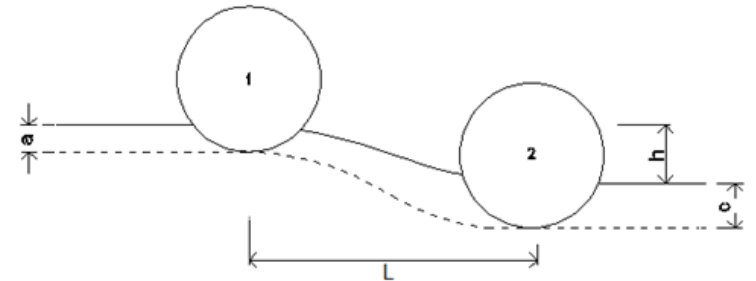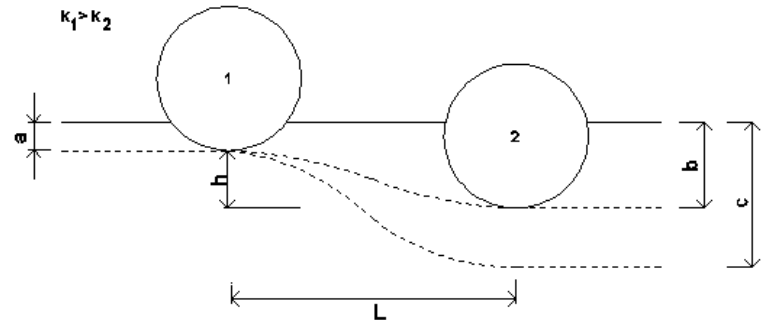# GENERATION OF VERTICAL ACCELERATION (a_v) DUE TO AN ASCENDING PROFILE
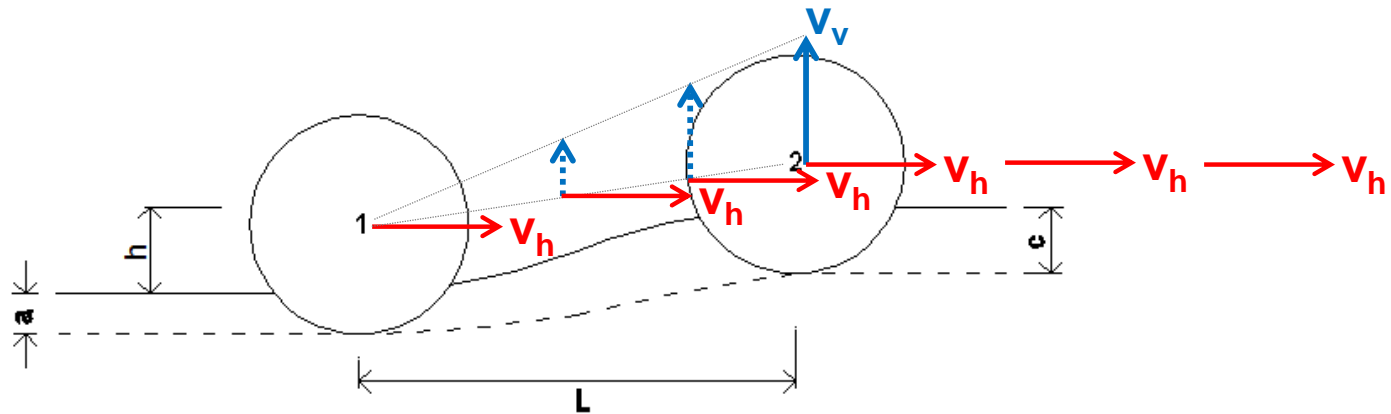
- As the wheel ascends along a transition due to increasing stiffness and/or an ascending track profile, the wheel tributary mass accelerates vertically (a_v) and reaches a vertical speed "v_v" at the end of the ascend.

- Once the cause of this vertical acceleration diminishes at the end of the ascend, the wheel **leaps onto** and drops on the track, generating an additional dynamic impact force represented by "K_j".

- This is **in addition** to the impact that develops due to the rate of change of potential energy (K'_{B2} and $K'_{B,a}$).
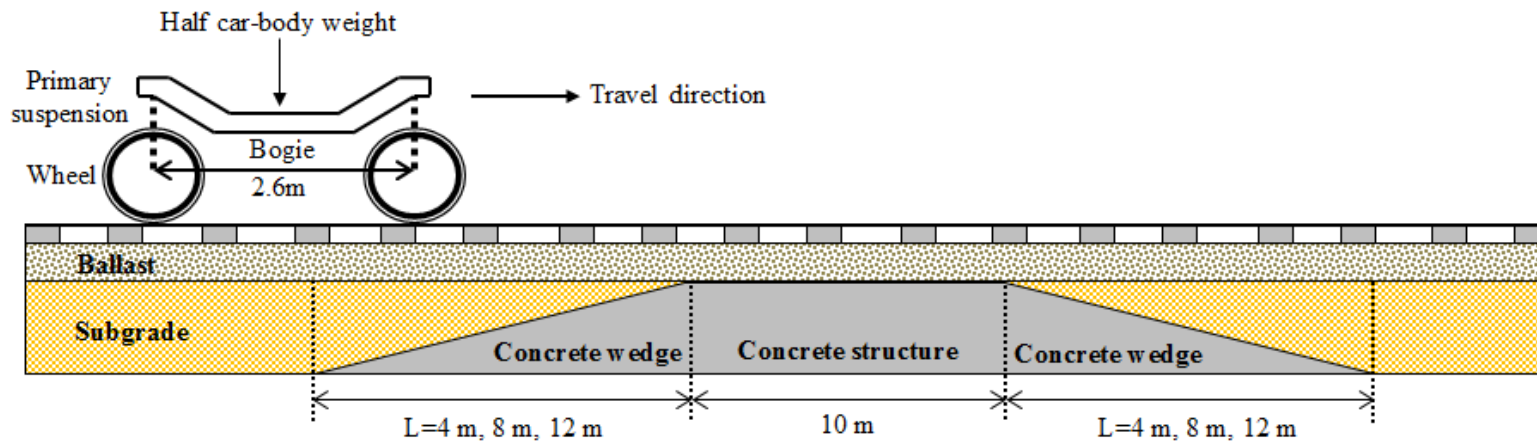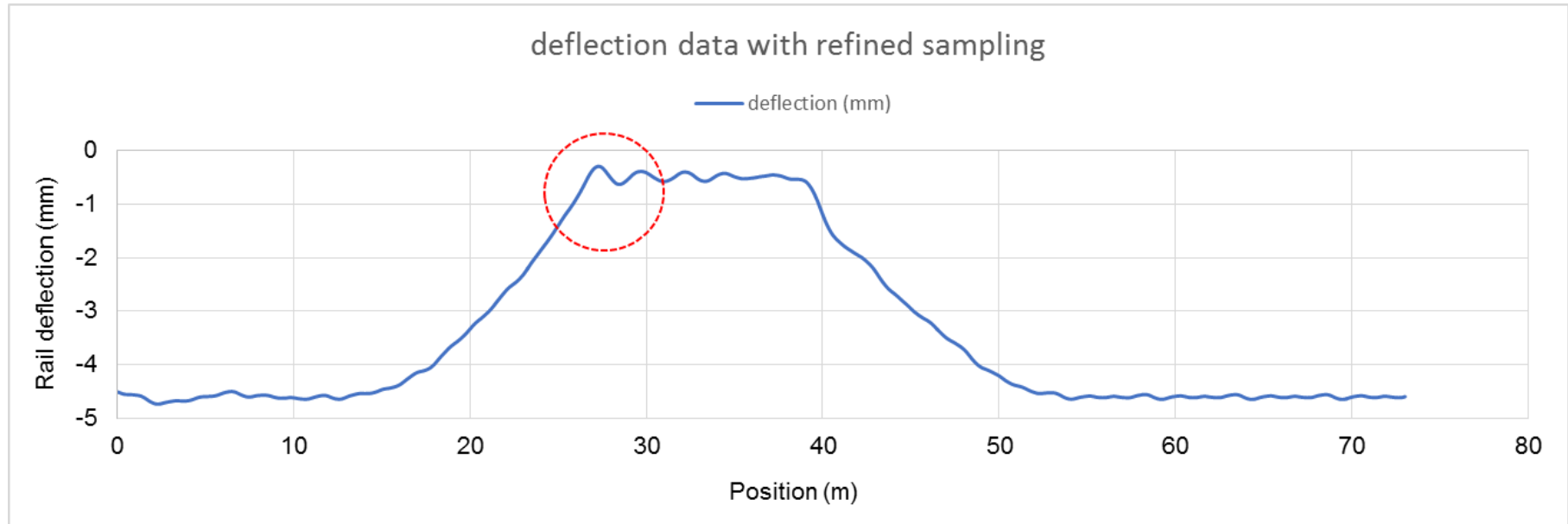
SECONDARY IMPACT K_J DUE TO WHEEL LEAP AT THE END OF AN ASCEND

$$h_a = \frac{v_v \cdot t}{2} = \frac{a_v \cdot t^2}{2} = \frac{a_v \cdot \left(\frac{L}{v}\right)^2}{2}$$

$$h_a = \frac{v_v \cdot t}{2} = \frac{a_v \cdot t^2}{2} = \frac{a_v \cdot \left(\frac{L}{v}\right)^2}{2}$$

$$a_v = 2h_a \cdot \left(\frac{v}{L}\right)^2$$

$$K_j = \frac{a_v}{g}$$

$k_1 < k_2$

Vertical speed

v_v

$h = h_a$

$t = \frac{L}{v}$

Time
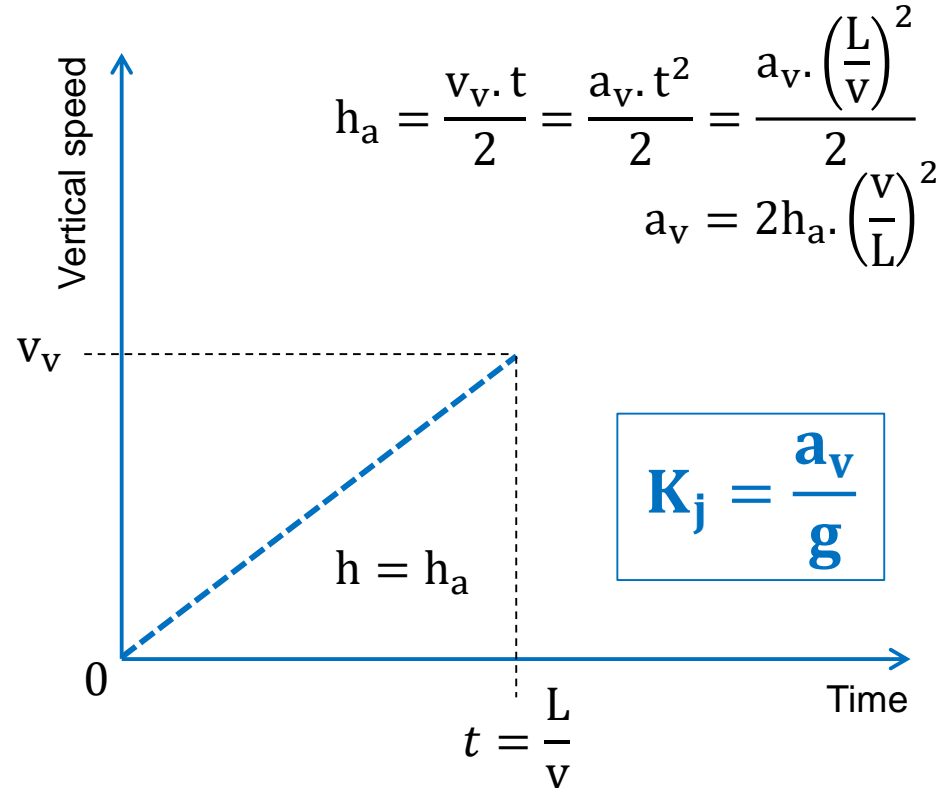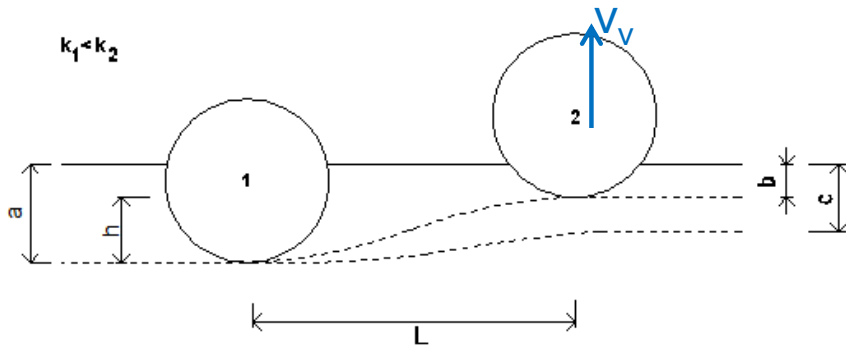
- K_j must be **added** to Bezgin equations for **ascending track profile and increasing track stiffness.**

- **Equivalent system stiffness does not reduce K_j.** $|a\text{-}b| = |a'\text{-}b'|$

- **FEA** estimates also indicate the presence of K_j.

APPLICATION

ANALYTICAL ASSESSMENT OF STIFFNESS TRANSITIONS:
AN ENVELOPE STUDY
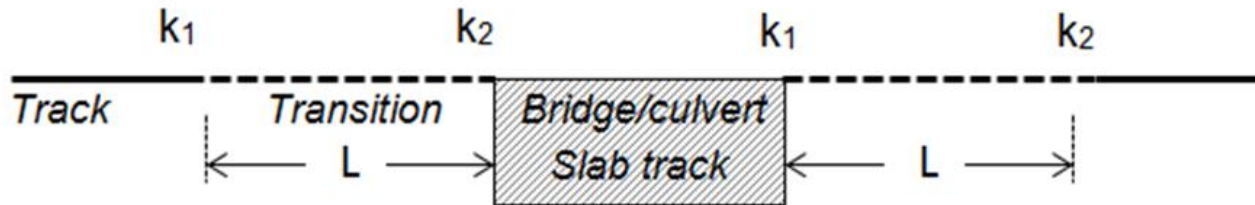
- Transition length varies from **L=2 m to 37 m** (7 ft to 120 ft).

- Train speed varies from **v=40 km/h to 241 km/h** (25 mph to 150 mph).

- Stiffness variation increases up to **5-fold** and decreases down to **1/5th**.

L -Transition track length-		Relative vertical stiffness ratio of structure (k_2, k_1) with respect to track (k_1, k_2)			v -Train speed-	
(m)	(ft)	k_1	k_2, k_1	k_2	(km/h)	(mph)
2	7					
4	13	1	1	1	40	25
6	20	1	1.5	1	80	50
12	40	1	2	1	121	75
18	60	1	3	1	161	100
24	80	1	4	1	201	125
30	100	1	5	1	241	150
37	120					

STIFFNESS VALUES FOR THE TRACK AND THE SYSTEM

- Track stiffness is **k=50 kN/mm** (286 kip/in). Representative values for the wheel, bogie and the Hertzian contact stiffness values are **k_w=3.6 kN/mm** (20.5 kip/in), **k_b=8.1 kN/mm** (46.2 kip/in) and **k_H=1600 kN/mm** (9,136.4 kip/in).

$$\frac{1}{k_{eq1}} = \frac{1}{k_b} + \frac{1}{k_w} + \frac{1}{k_H} + \frac{1}{k_1} = \frac{1}{8.1} + \frac{1}{3.6} + \frac{1}{1600} + \frac{1}{50} \rightarrow k_{eq1} = 2.37 \text{ kN/mm}$$

- The wheel sees a substantially lower equivalent system stiffness than track stiffness .

- Static wheel force of a passenger train is **F_s= 90 kN** (20,232 lb).

- The combined **system deflection** under F_s is **a'=F_s/k_{eq1}=37.97 mm** (1.49 in).

- **Track deflection** is **a=F_s/k_1=90 kN / 50 kN/mm = 1.8 mm.**

- The **remaining 36.17 mm** distributes to wheel, bogie and Hertzian spring deflections.

EQUIVALENT SYSTEM STIFFNESS DUE TO *3-FOLD* STIFFNESS TRANSITIONS

- Let's consider the case of **$k_2=3k_1=150$ kN/mm (856 kip/in)** where the stiffness increases **three-fold** as train enters the structure.

- The system deflection at region-2 is **$b'=F_s/k_{eq2}=36.77$ mm** (1.45in), the track deflection is **$b=F_s/k_1=90$ kN / 150 kN/mm = 0.6 mm**.

$$\frac{1}{k_{eq2}} = \frac{1}{k_b} + \frac{1}{k_w} + \frac{1}{k_H} + \frac{1}{k_1} = \frac{1}{8.1} + \frac{1}{3.6} + \frac{1}{1600} + \frac{1}{150} \rightarrow k_{eq2} = 2.45 \text{ kN/mm}$$

- The resulting elevation differential that results in a potential energy difference for the tributary wheel mass is:
  h= a'- b'= 37.97-36.77 = 1.2 mm (0.05 in).

- If one excludes the wheel and bogie springs and considers the stiffness of the track only, the net change in the wheel elevation would naturally be the same:
  h=a-b=1.8 mm − 0.6 mm= 1.2 mm (0.047 in).

ESTIMATED DYNAMIC IMPACT FORCE FACTORS DUE TO *3-FOLD* TO *1/3RD* STIFFNESS TRANSITIONS FOR L=6 M *(20 FT)* AT V=241 KM/H *(150 MPH)*

$$f = 1 - \frac{t_{fall}}{t_{pass}} = 1 - \frac{\sqrt{2 \cdot h / g}}{L / v} = 1 - \frac{v}{L} \cdot \sqrt{\frac{2h}{g}} = 1 - \frac{66.94 \frac{m}{s}}{6 \text{ m}} \cdot \sqrt{\frac{2 * 0.0012 m}{9.81 \frac{m}{s^2}}} = 0.825$$

$$K_{B2} = \sqrt{2 \left[1 + f + \frac{a}{b} \cdot (1 - f)\right]} - 1 = \sqrt{2 \left[1 + 0.825 + \frac{1.8}{0.6} \cdot (1 - 0.825)\right]} - 1 = 1.168$$

$$K_{B1} = 1 + \sqrt{2 \left[(1 - f) \cdot \left(1 - \frac{a}{b}\right)\right]} = 1 + \sqrt{2 \left[(1 - 0.825) \cdot \left(1 - \frac{0.6}{1.8}\right)\right]} = 1.483$$
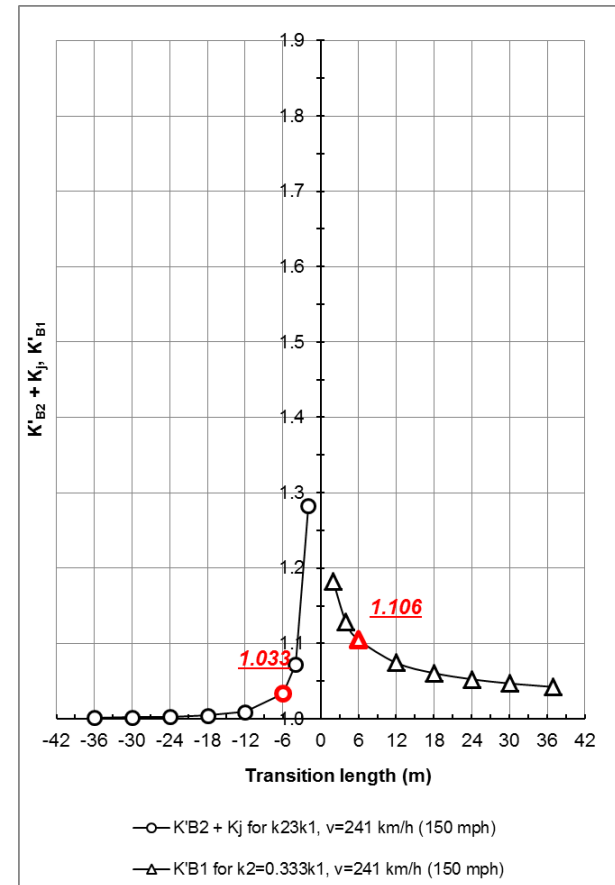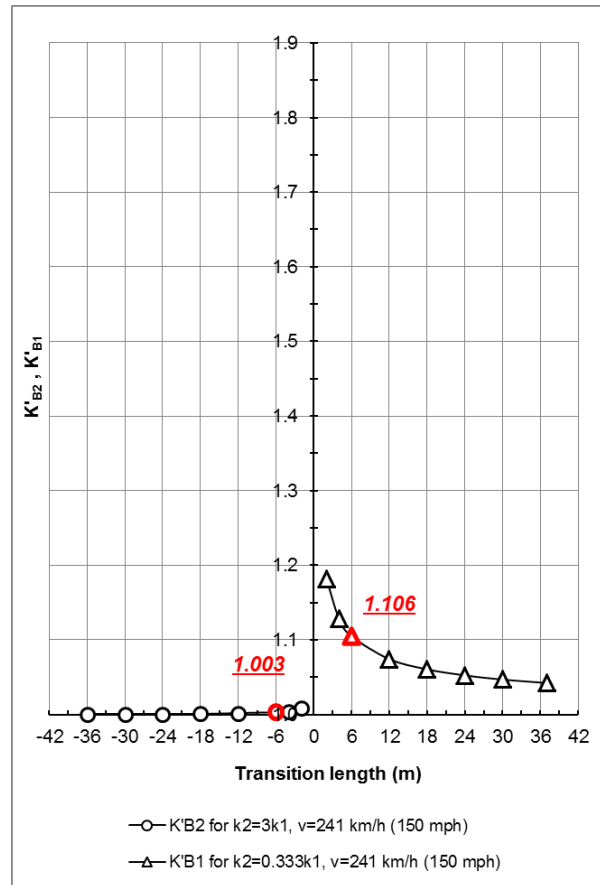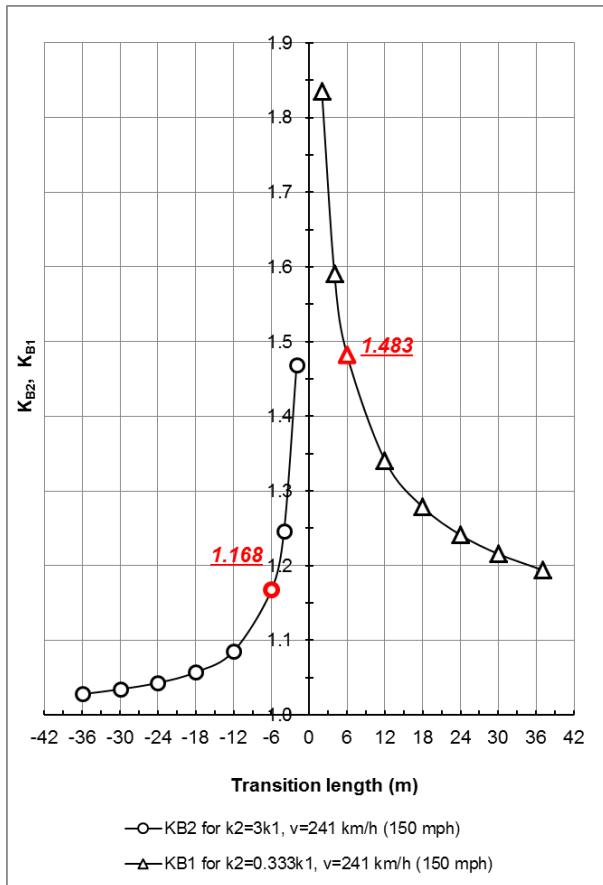
$$K'_{B2} = \sqrt{2 \left[1 + 0.825 + 0 + \frac{37.97}{36.77} (1 - 0.825 - 0)\right]} - 1 = 1.003$$

$$K'_{B1} = 1 + \sqrt{2 \left[(1 - 0.825 - 0) \cdot \left(1 - \frac{36.77}{37.97}\right)\right]} = 1.106$$
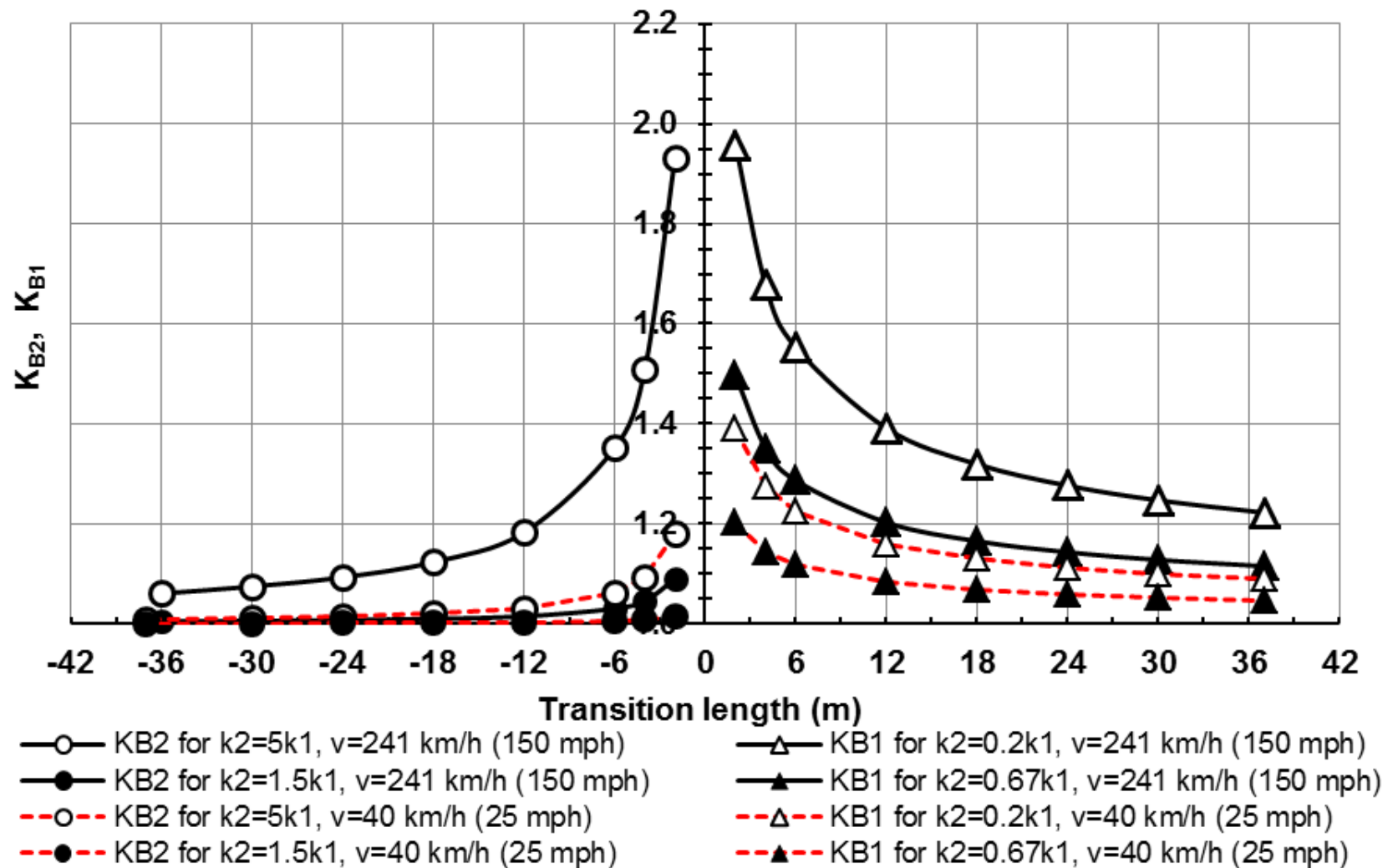
K'_{B2} + K_j = 1.033

$$K_j = \frac{2h_a}{g} \cdot \left(\frac{v}{L}\right)^2 = \frac{2 * 0.0012}{9.81} \cdot \left(\frac{66.94}{6}\right)^2 = 0.03046$$

ESTIMATED DYNAMIC IMPACT FORCE FACTORS DUE TO *3-FOLD* TO *1/3RD* STIFFNESS TRANSITIONS FOR L=6 M *(20 FT)* AT V=241 KM/H *(150 MPH)*
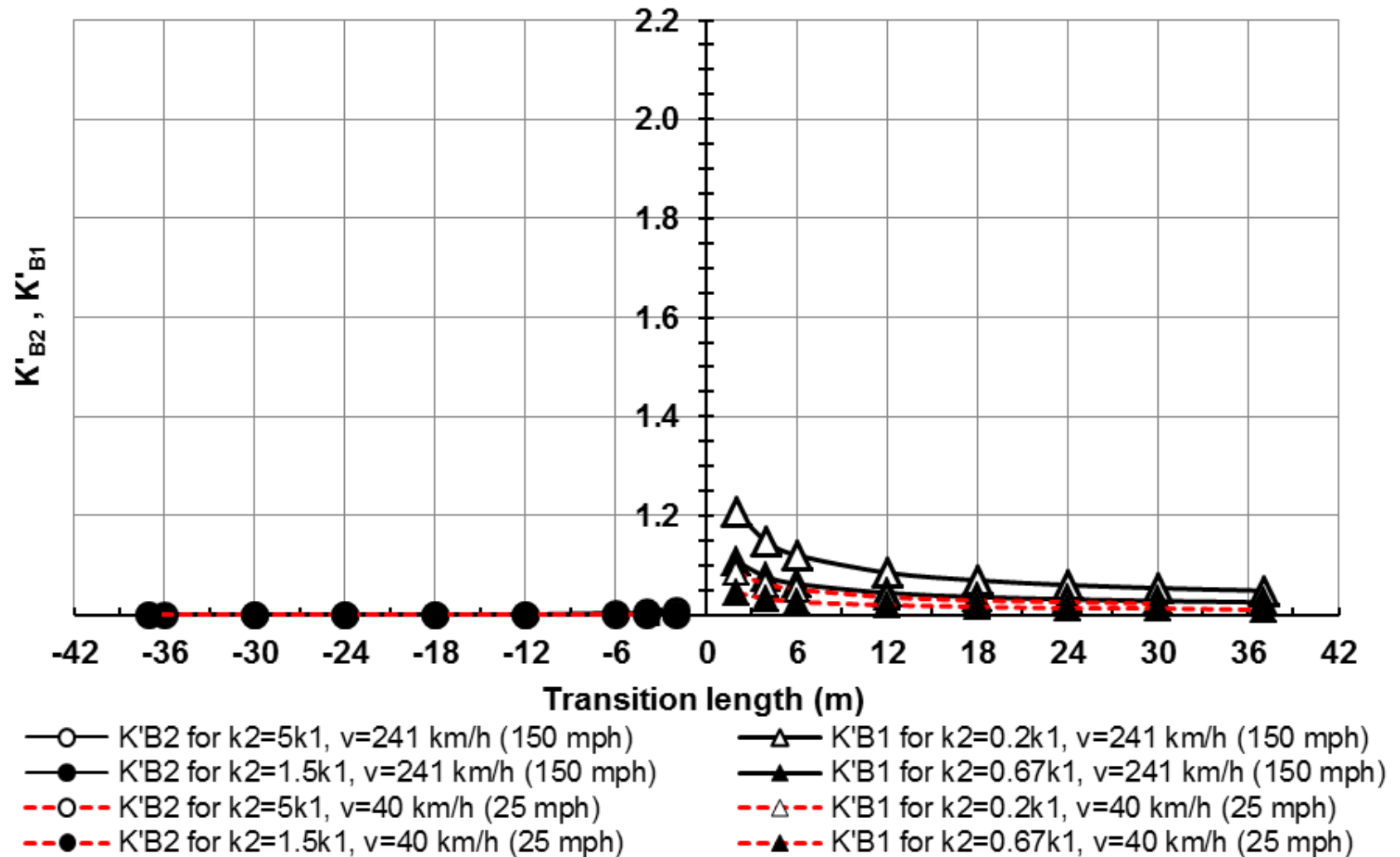

- Each estimate can be attained with a manual calculation that lasts **less than a minute**.

KB2 for k2=5k1, v=241 km/h (150 mph)
KB2 for k2=1.5k1, v=241 km/h (150 mph)
KB2 for k2=5k1, v=40 km/h (25 mph)
KB2 for k2=1.5k1, v=40 km/h (25 mph)

KB1 for k2=0.2k1, v=241 km/h (150 mph)
KB1 for k2=0.67k1, v=241 km/h (150 mph)
KB1 for k2=0.2k1, v=40 km/h (25 mph)
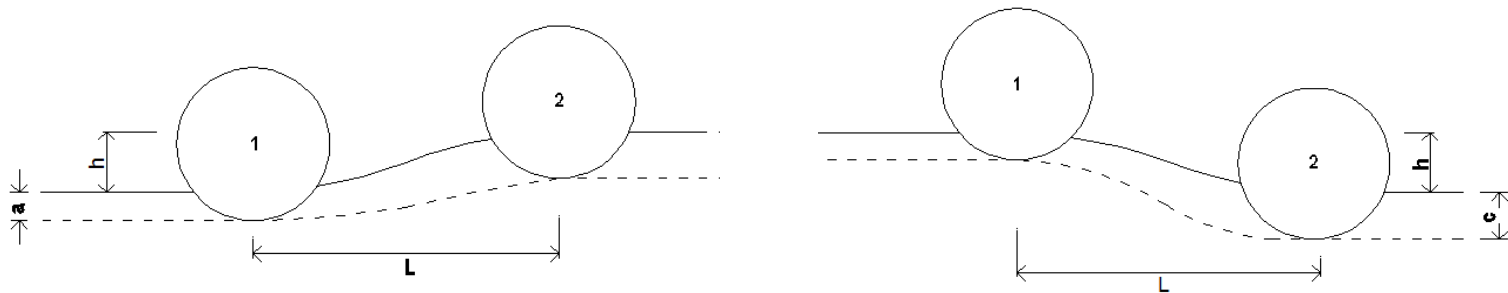KB1 for k2=0.67k1, v=40 km/h (25 mph)

ESTIMATES OF EXTENDED BEZGIN EQUATIONS: $(K'_{B2} + K_J)$ AND K'_{B1} UP TO *5-FOLD* TO $1/5^{TH}$ STIFFNESS TRANSITIONS_CONSİDERİNG THE EQUIVALENT SYSTEM STIFFNESS
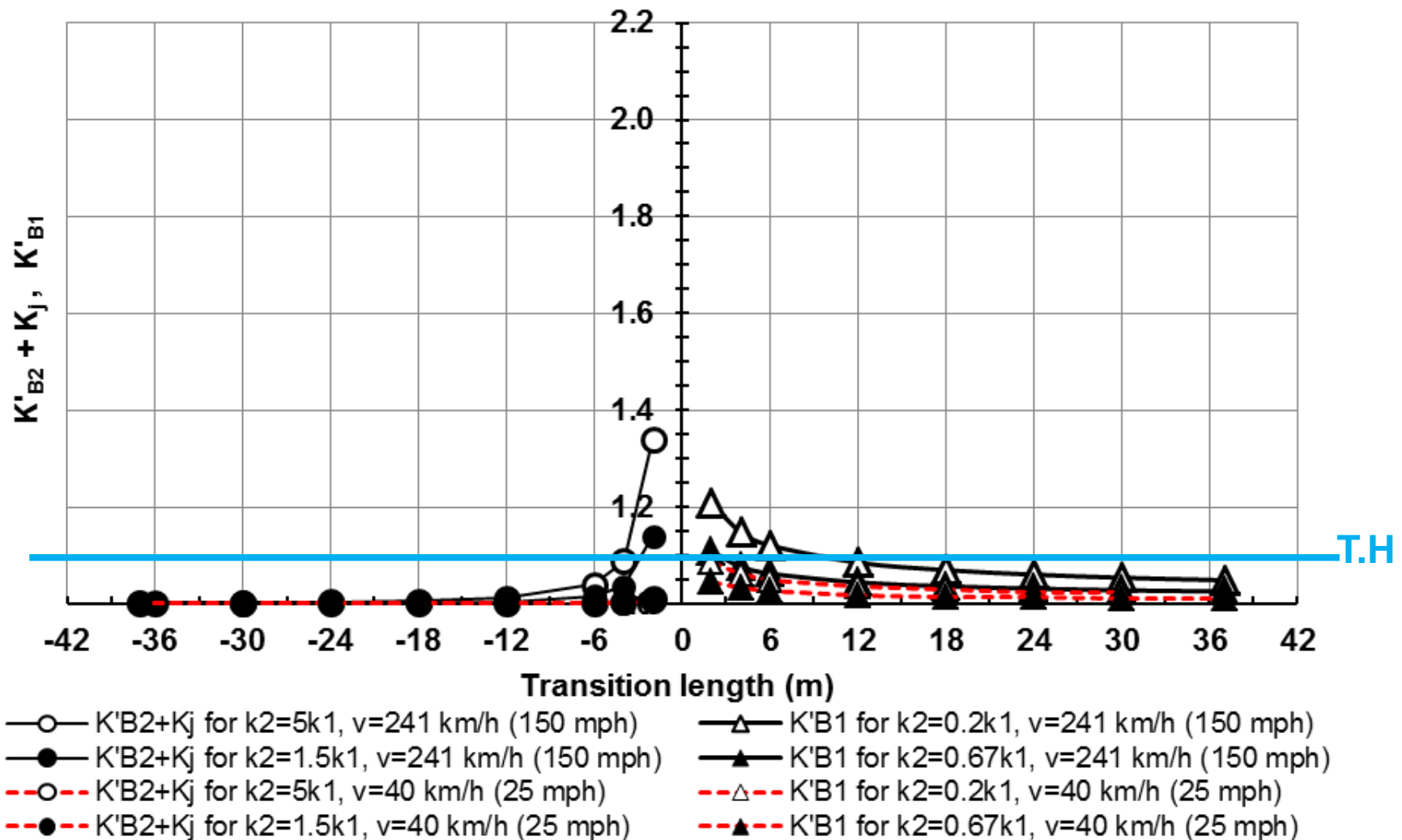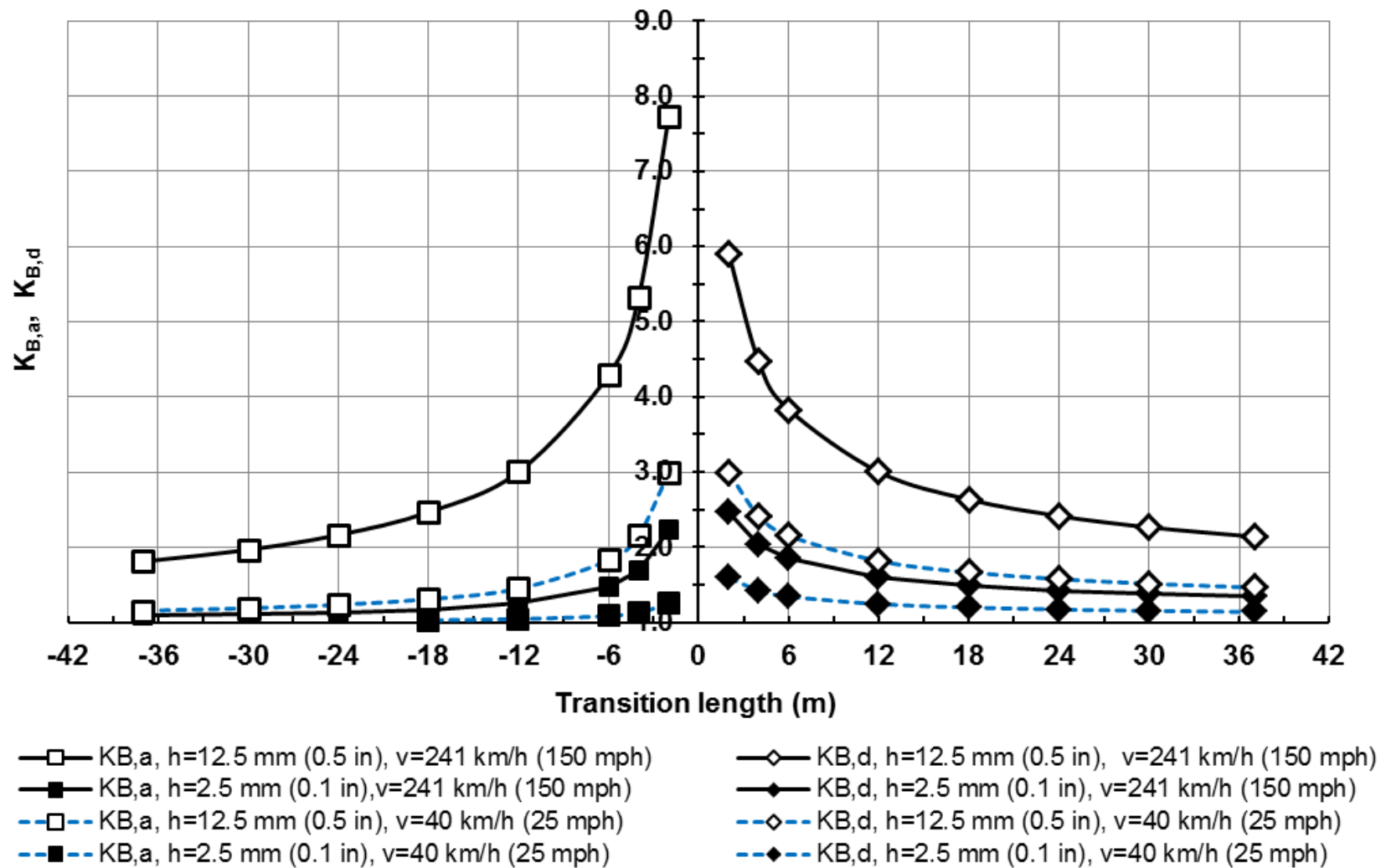
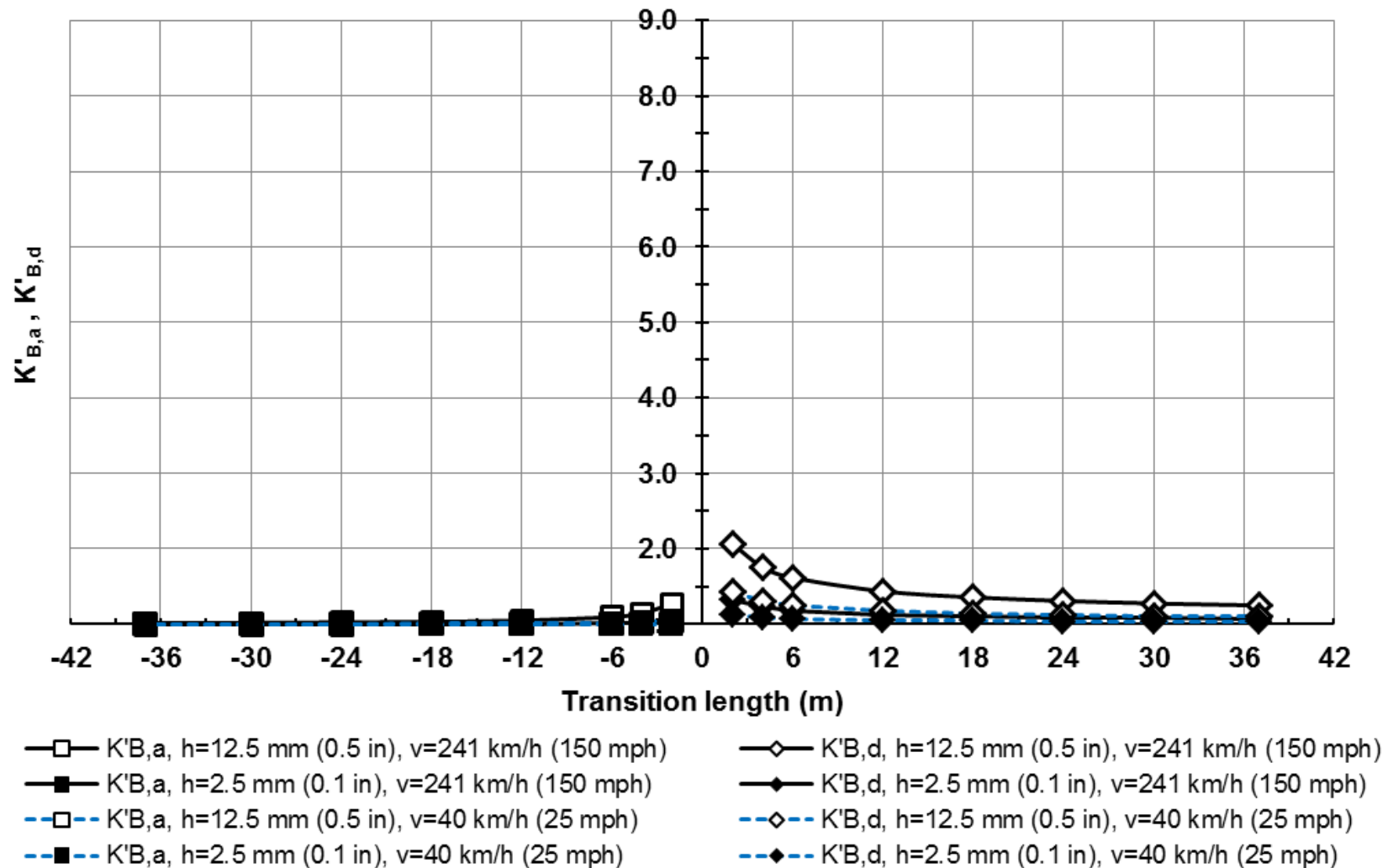## ANALYTICAL ASSESSMENT OF PROFILE VARIATIONS:
AN ENVELOPE STUDY

- Transition length varies from **L=2 m to 37 m** (7 ft to 120 ft).

- Train speed varies from **v=40 km/h to 241 km/h** (25 mph to 150 mph).

- Minimum value of track profile deviation of **h=2.5 mm** (0.1 in) and maximum value of **h=12.5 mm** (0.5 in)

- Track stiffness is **k=50 kN/mm** (286 kip/in).

- Static wheel force of a passenger train is **F_s= 90 kN** (20,232 lb).

- Representative values for the wheel, bogie and the Hertzian contact stiffness values are **k_w=3.6 kN/mm** (20.5 kip/in), **k_b=8.1 kN/mm** (46.2 kip/in) and **k_H=1600 kN/mm** (9,136.4 kip/in).
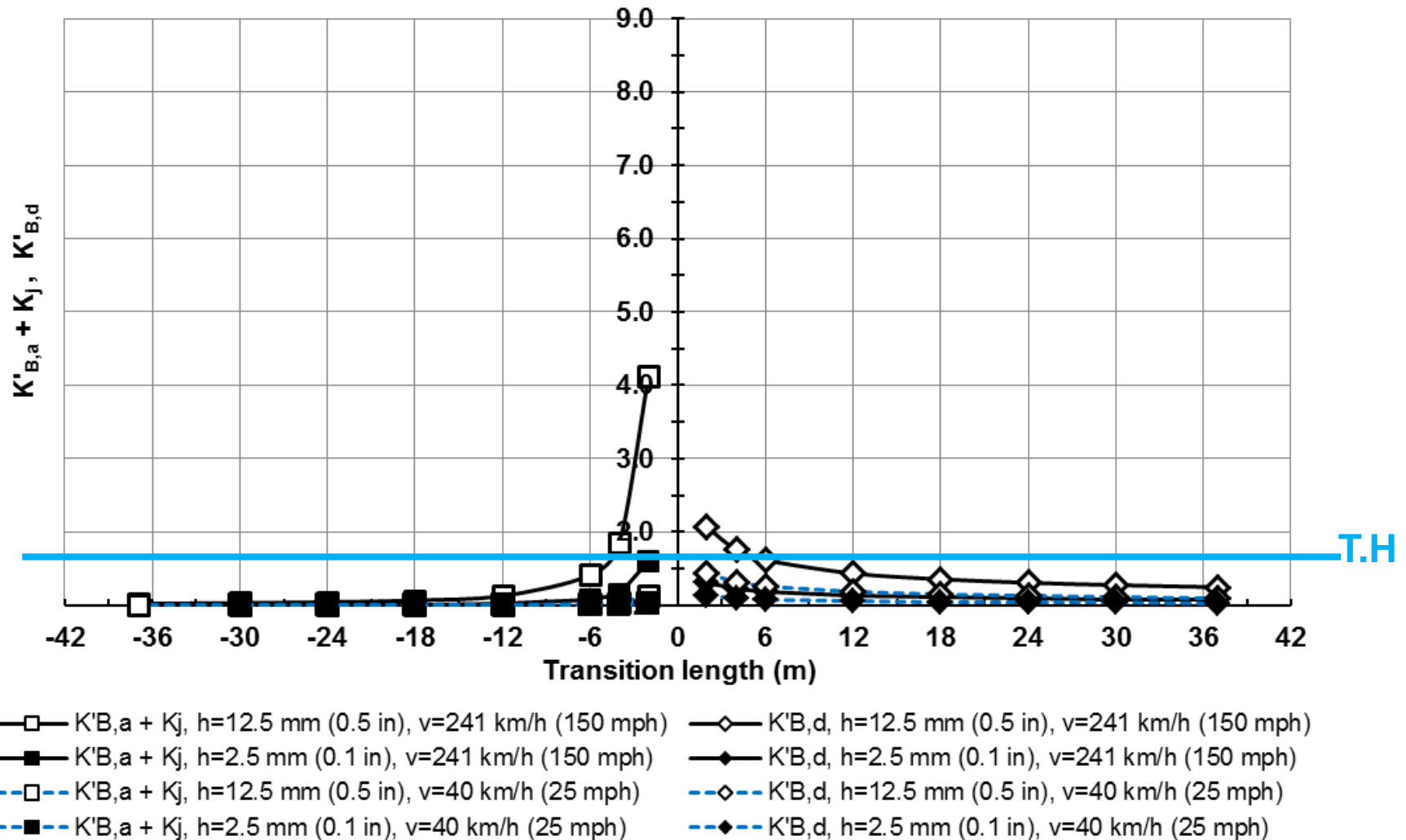
BEZGIN EQUATIONS: $K_{B,a}$ and $K_{B,d}$

Legend:
- KB,a, h=12.5 mm (0.5 in), v=241 km/h (150 mph)
- KB,a, h=2.5 mm (0.1 in), v=241 km/h (150 mph)
- KB,a, h=12.5 mm (0.5 in), v=40 km/h (25 mph)
- KB,a, h=2.5 mm (0.1 in), v=40 km/h (25 mph)
- KB,d, h=12.5 mm (0.5 in), v=241 km/h (150 mph)
- KB,d, h=2.5 mm (0.1 in), v=241 km/h (150 mph)
- KB,d, h=12.5 mm (0.5 in), v=40 km/h (25 mph)
- KB,d, h=2.5 mm (0.1 in), v=40 km/h (25 mph)

EXTENDED BEZGIN EQUATIONS: $K'_{B,a}$ and $K'_{B,d}$

# EXTENDED BEZGIN EQUATIONS: $(K'_{B,a} + K_j)$ and $K'_{B,d}$

TOTAL DYNAMIC IMPACT FORCE FACTORS at ENTRANCE and at EXIT

- in the presence of **stiffness transition** and **profile variation**, the user must consider the developing dynamic impact forces due to both variations.

$$K'_{ENTER} = K'_{B2} + K'_{B,a} + K_j - 1$$

$$K'_{EXIT} = K'_{B1} + K'_{B,d} - 1$$

TOTAL DYNAMIC IMPACT FACTORS AT THE ENTRANCE AND EXIT DUE TO *3-FOLD TO 1/3RD* STIFFNESS TRANSITION AND PROFILE VARIATION OF *h=2.5 MM* AND *12.5 MM (0.5 İN)* FOR A SPEED OF *v=241 KM/H (150 MPH)*
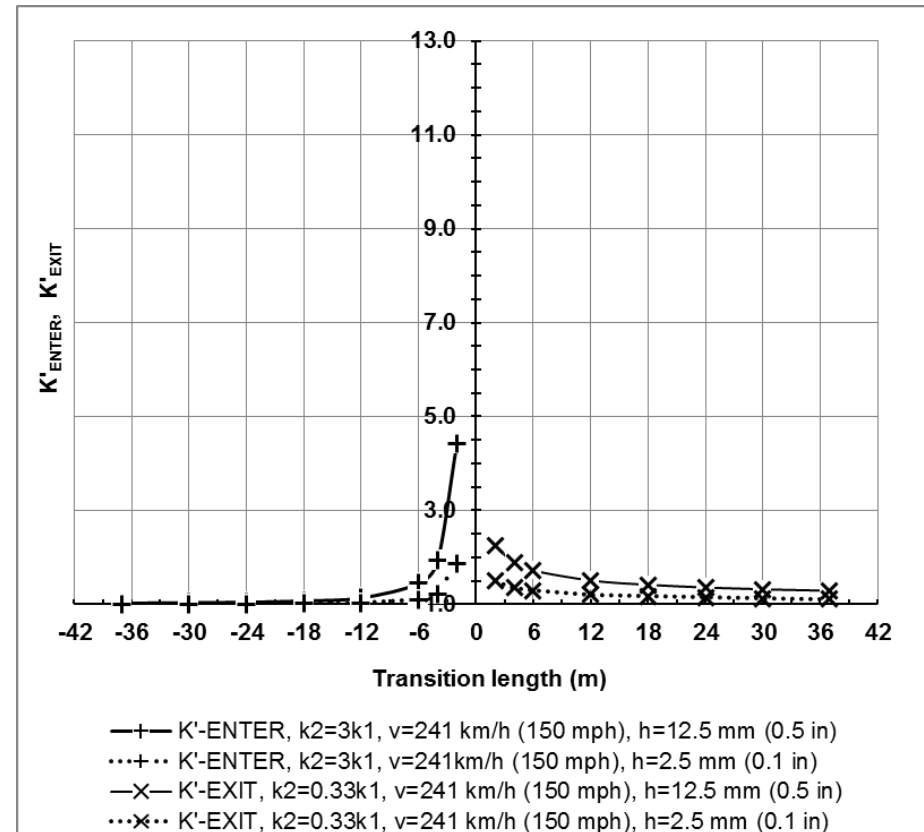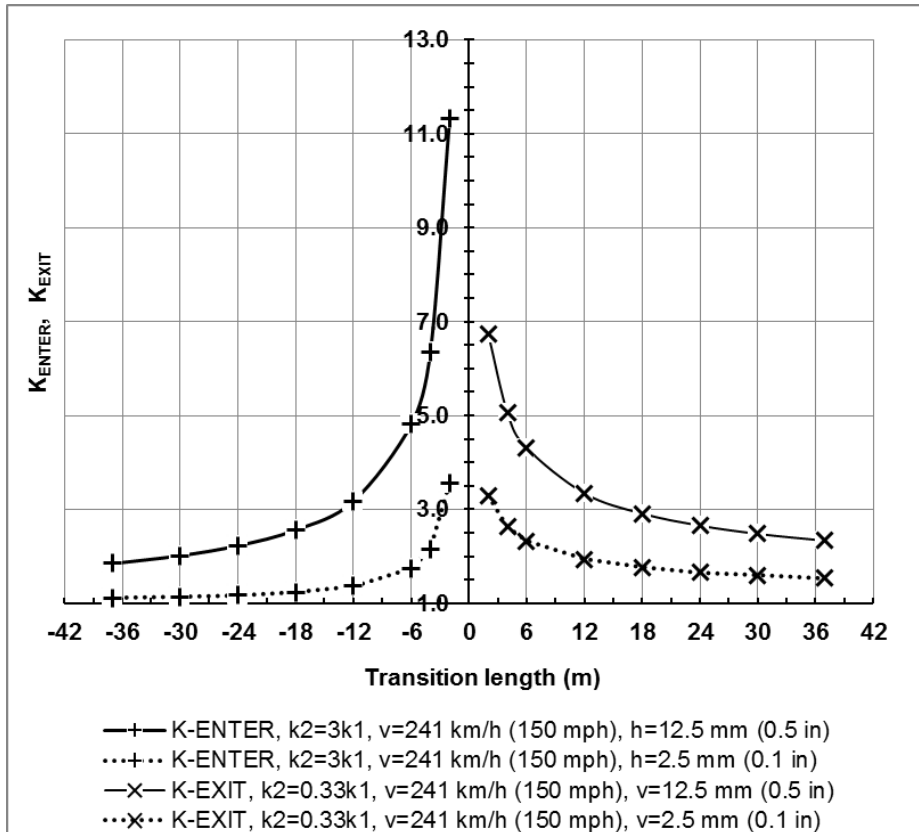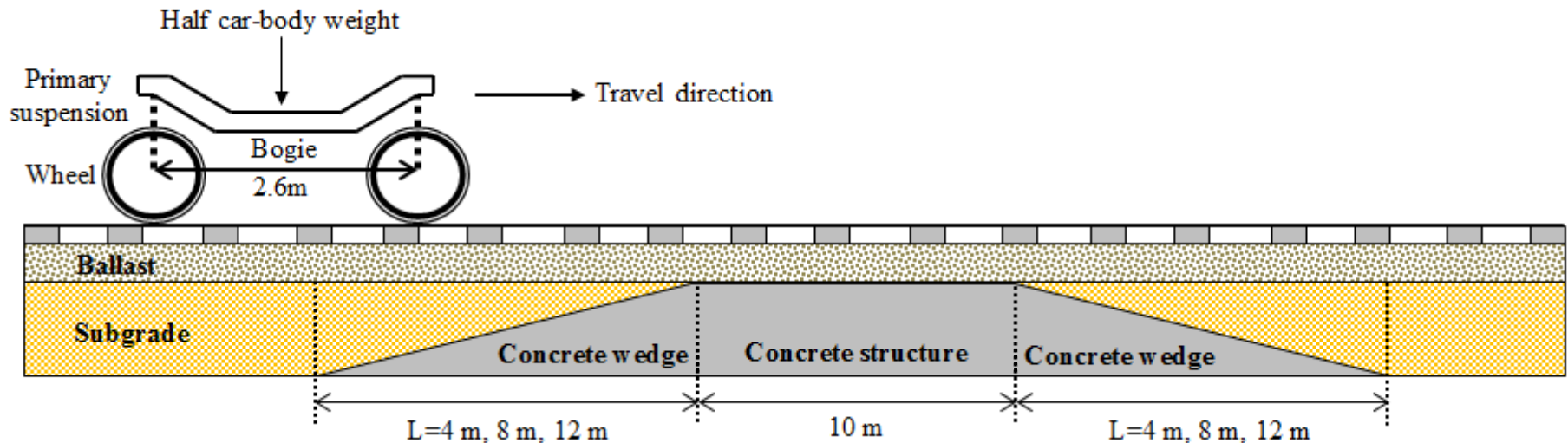

6 m = 20 ft

COMPARISON BY FEM RESULTS

*Model developed by DR.MOHAMED WEHBI from NETWORK RAIL*

CONCEPTUAL MODEL OF A TRACK STIFFNESS TRANSITION

- The model has a total length of **70 m** (230 ft).

- A train bogie travels, at speeds **v=120 kph** (75mph) and **v=250 kph** (155 mph).

- The wheel exerts a static force **F_s=68.6 kN** (15.4 kip) on the track.

- Ballast track resting on **soft subgrade soil** transitioning along **L=4 m, 8 m, 12 m** (13 ft, 26 ft, 39 ft) to a ballast track resting on a 10 m long (33 ft) **concrete structure**.

Finite element model of track transition developed with ABAQUS®
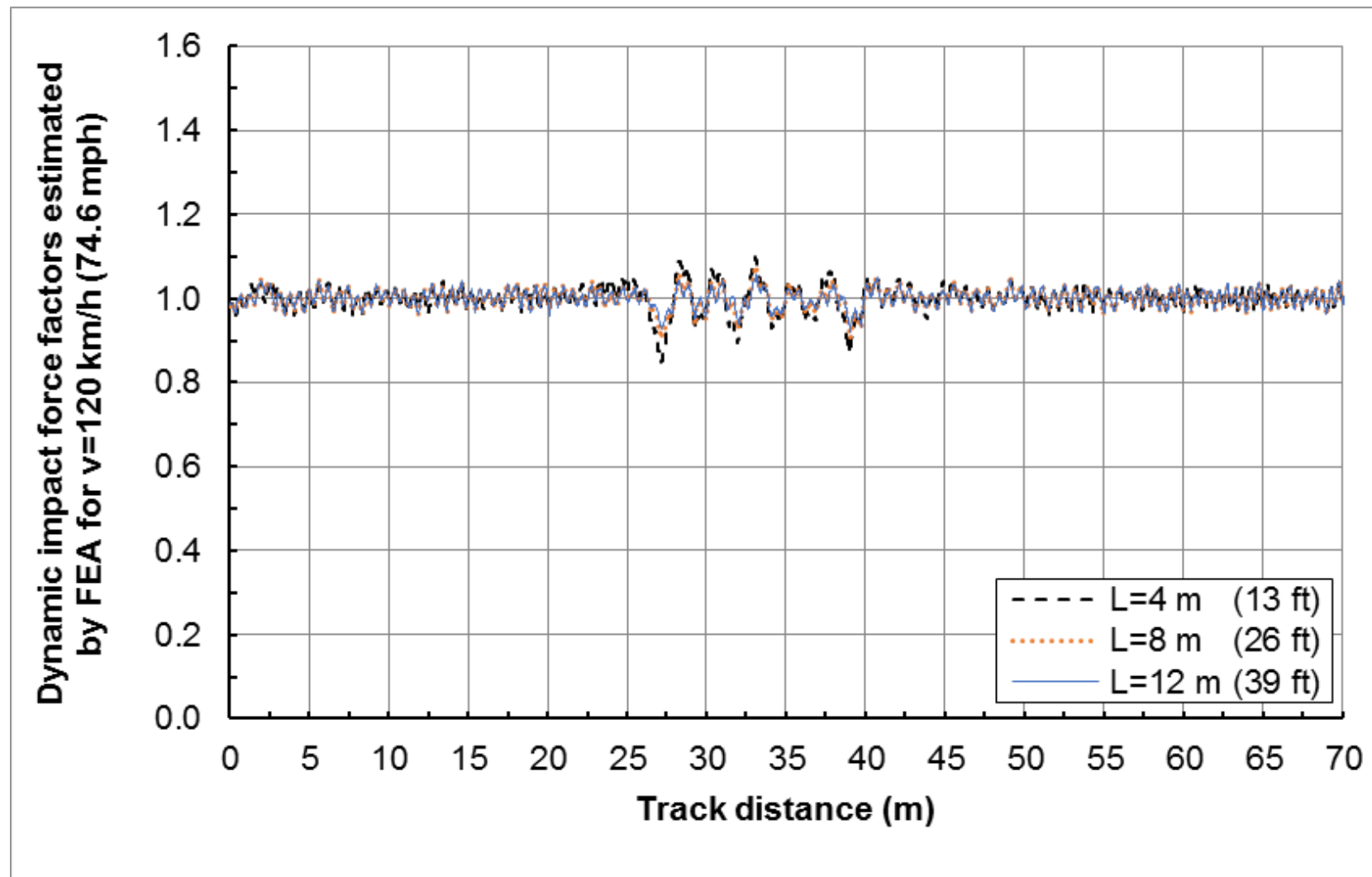# Model developed by DR. MOHAMED WEHBI from NETWORK RAIL
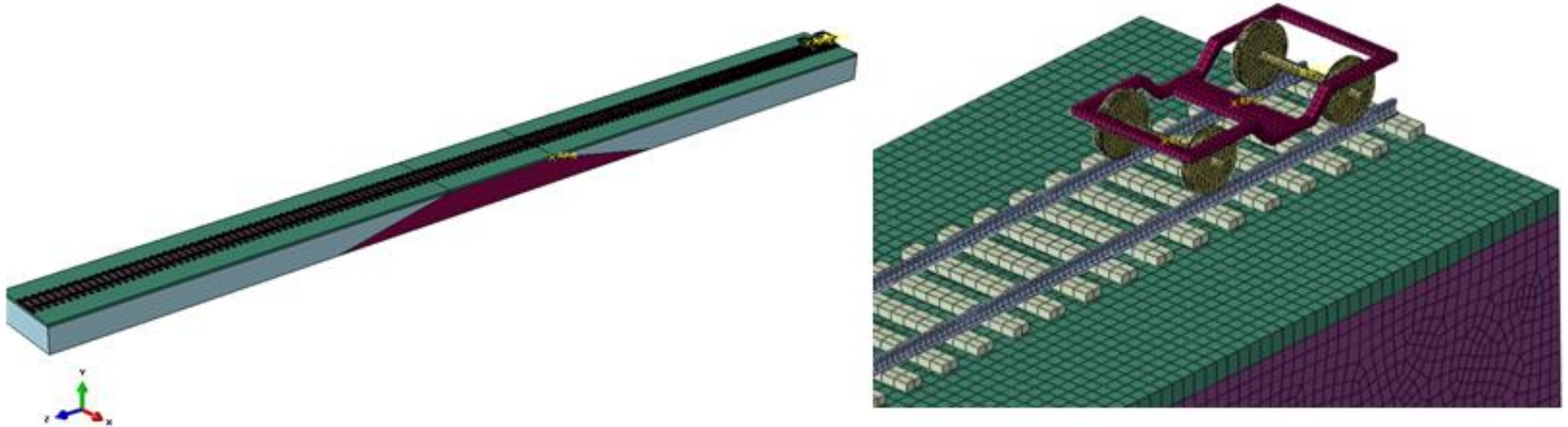
Total length (m)	70	Hertzian contact spring (N/m)	10×10^9
Train speed (kph)	120, 250	Ballast Young's Modulus (MPa)	180
Total axle load (kN)	137.2	Ballast thickness (mm)	300
Wheel diameter (m)	0.9	Ballast density (kg/m^3)	1800
Wheel set mass (kg)	981	Ballast Poisson ratio	0.28
Bogie mass (kg)	2707	Ballast loss factor	0.1
Primary suspension stiffness (N/m)	0.179×10^6	Rail Pad stiffness (kN/mm)	150
Primary suspension damping (N.s/m)	4200	Rail Pad loss factor	0.2
Half of Carbody (kg)	10700	Subgrade Young's modulus (MPa)	10
Rail type	CEN60	Subgrade thickness (mm)	3000
Rail Young's modulus (MPa)	210,000	Subgrade density (kg/m^3)	2000
Rail density (kg/m^3)	7750	Subgrade Poisson ratio	0.3
Rail loss factor	0.01	Subgrade loss factor	0.5
Sleeper dimensions (m) (length x width x height)	2.5x0.25x0.14	Concrete Young's modulus (MPa)	35000
Sleeper spacing (m)	0.6	Concrete density (kg/m^3)	2400

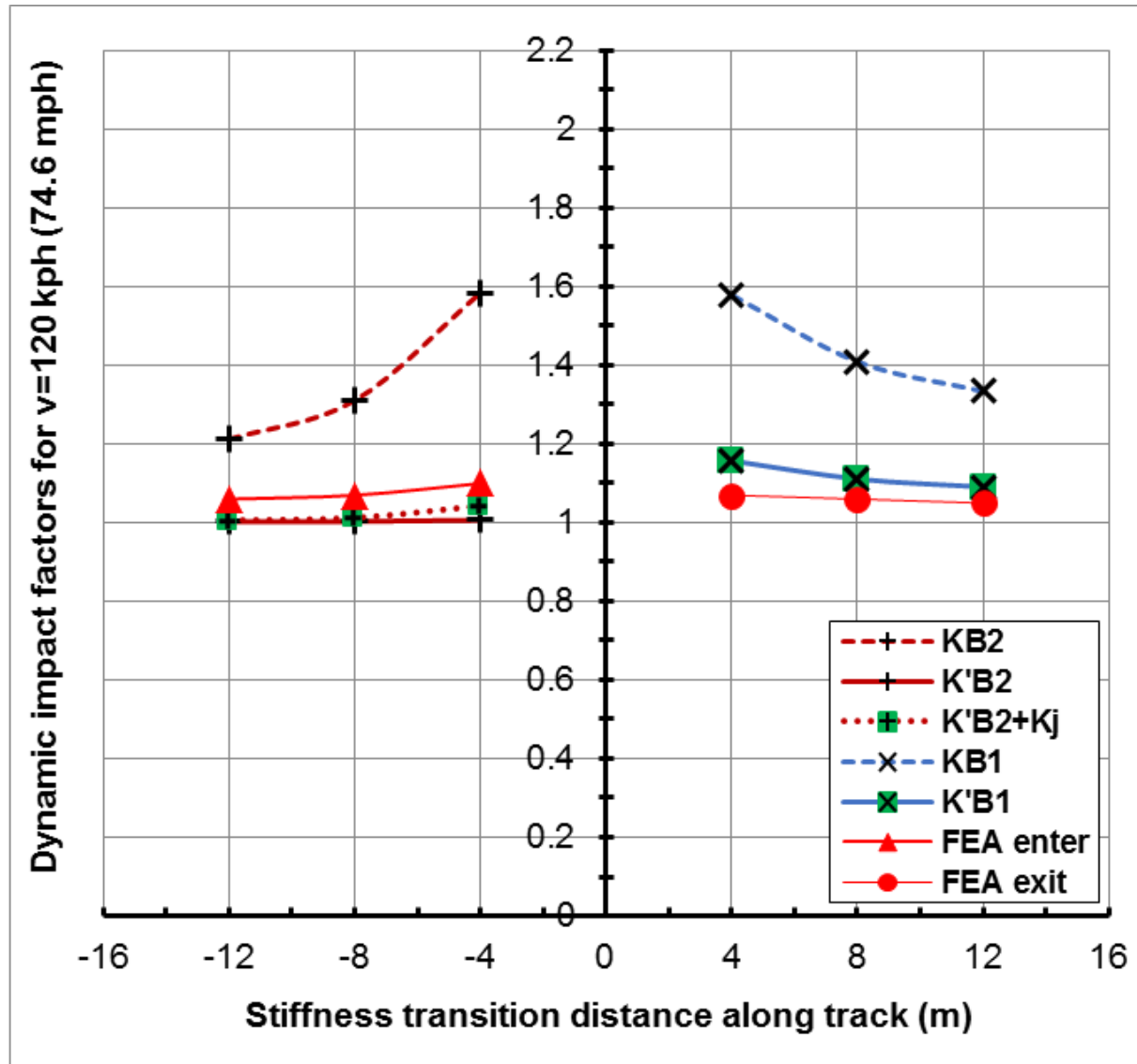k_{eq1}=1.71 kN/mm (9.75 kip/in) and k_{eq2}=1.83 kN/mm (10.45 kip/in)

GIVEN FEM INFORMATION

- Count- 309,613 elements, 16 core supercomputer of Network Rail

- More than **24 hours** running time for **each** simulation, output file for each analysis: 15 Gb

- More than **two weeks** to set up a model: 1 engineer with a Ph.D degree and 1 assistant, **6 hours** required to set-up for each variation of a model

- Total of **9 runs** on four seperate models completed for this study.

- FEA time: 9 * 24 hours = **9 days**

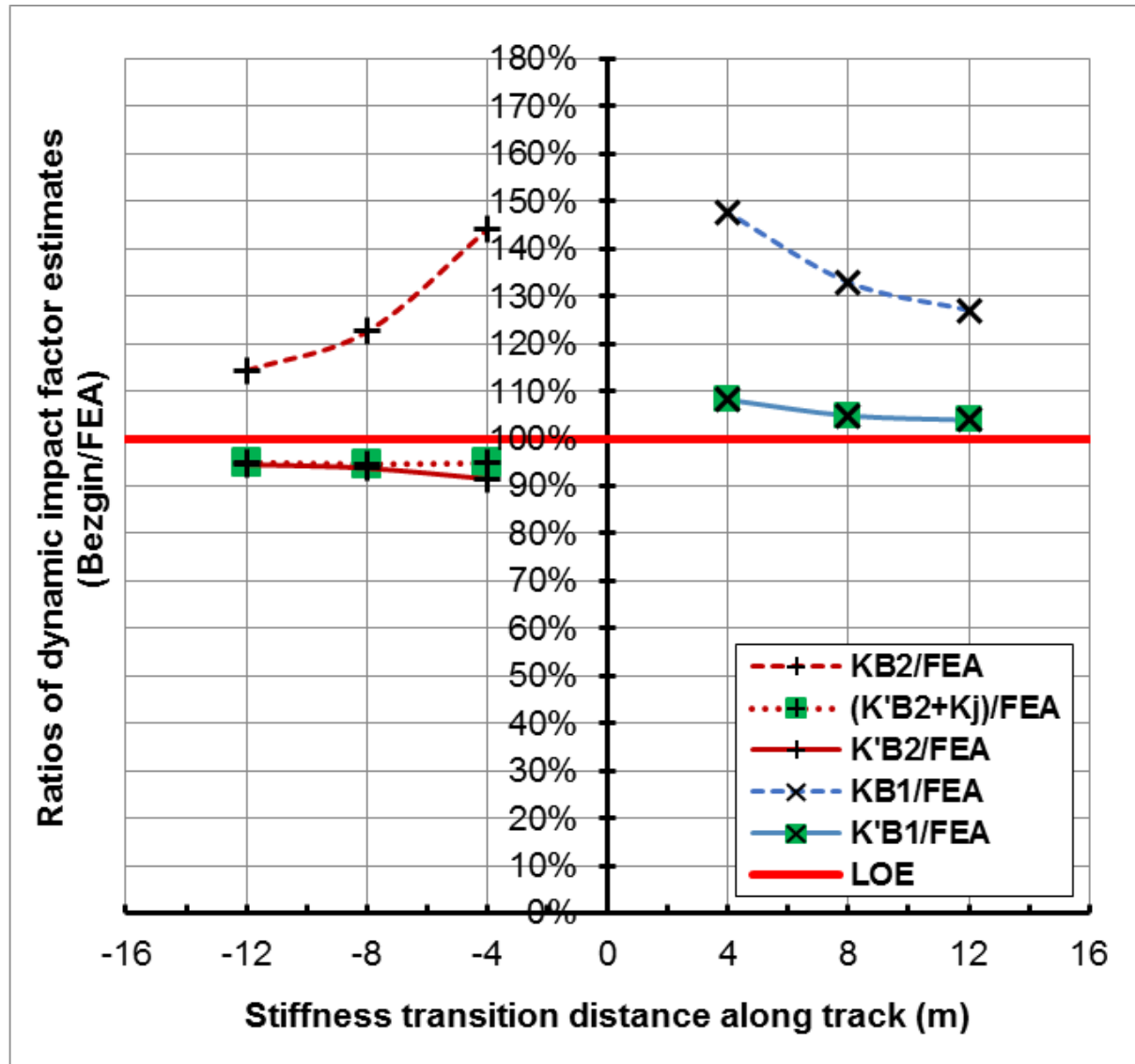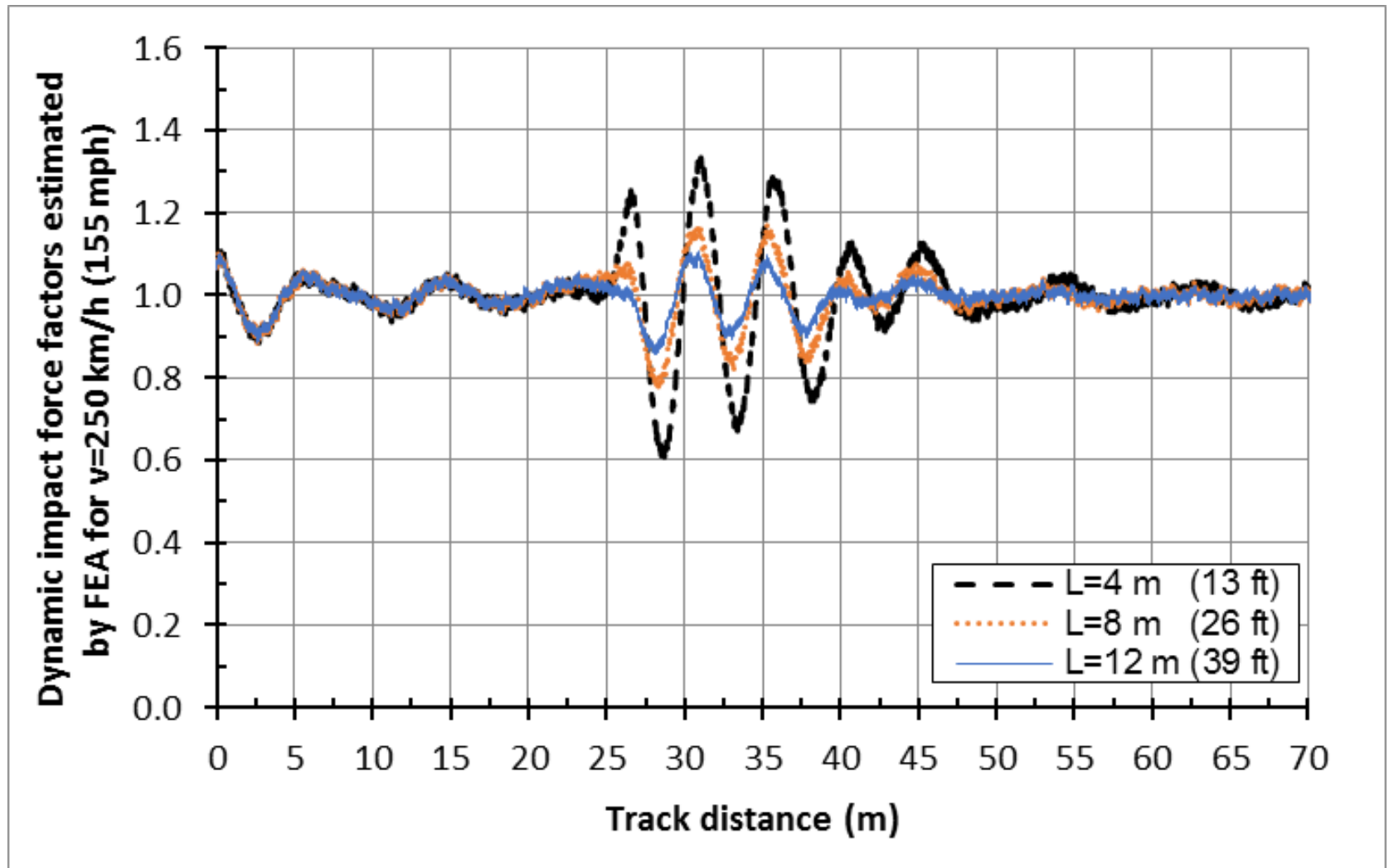# COMPARISON OF BEZGIN METHOD AND FEA ESTIMATES FOR DYNAMIC IMPACT FORCE FACTORS_L= 4 M, 8 M, 12 M *(13 FT, 26 FT, 39 FT)_V=75 MPH_*FEA LASTED 3 DAYS
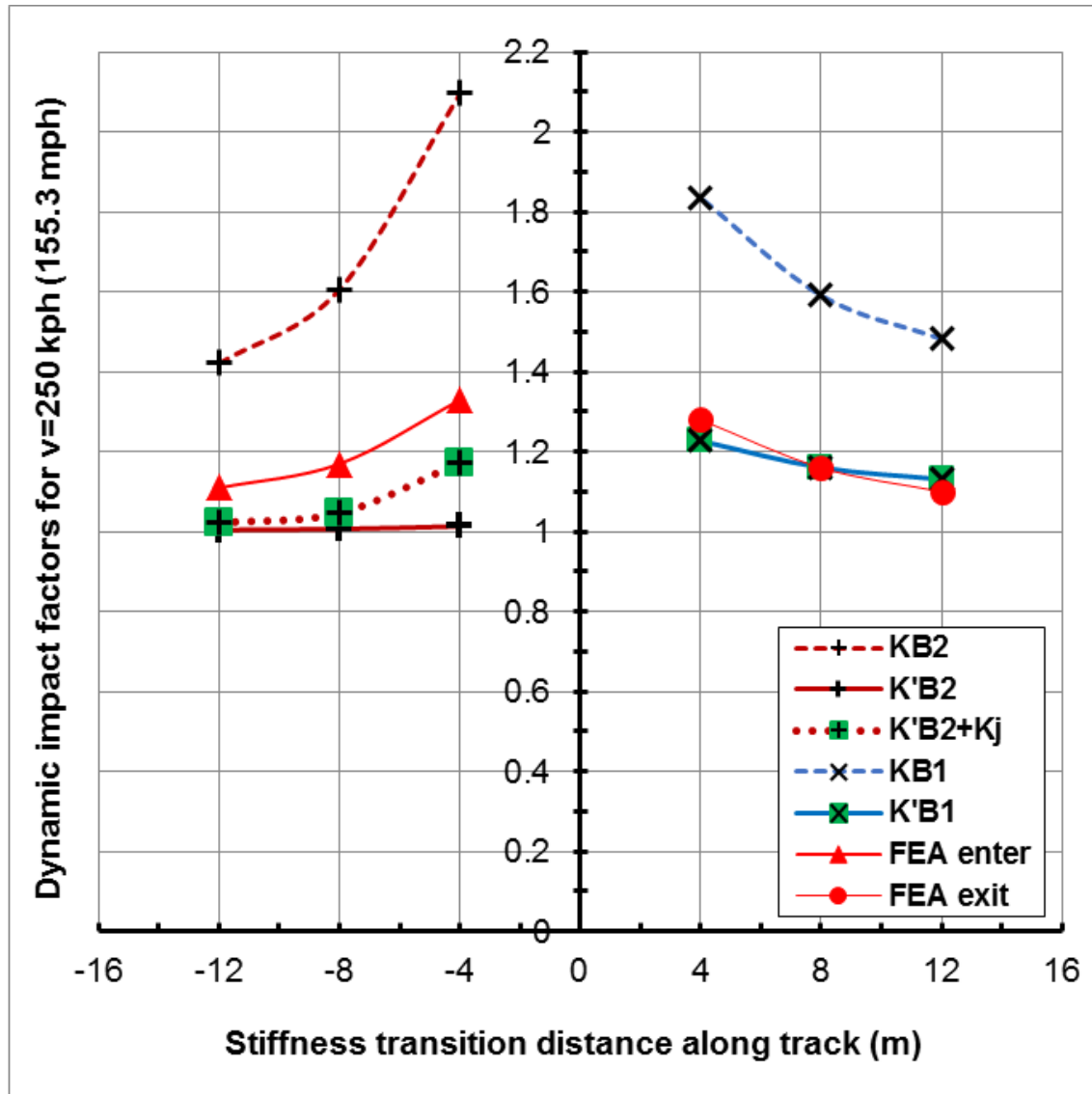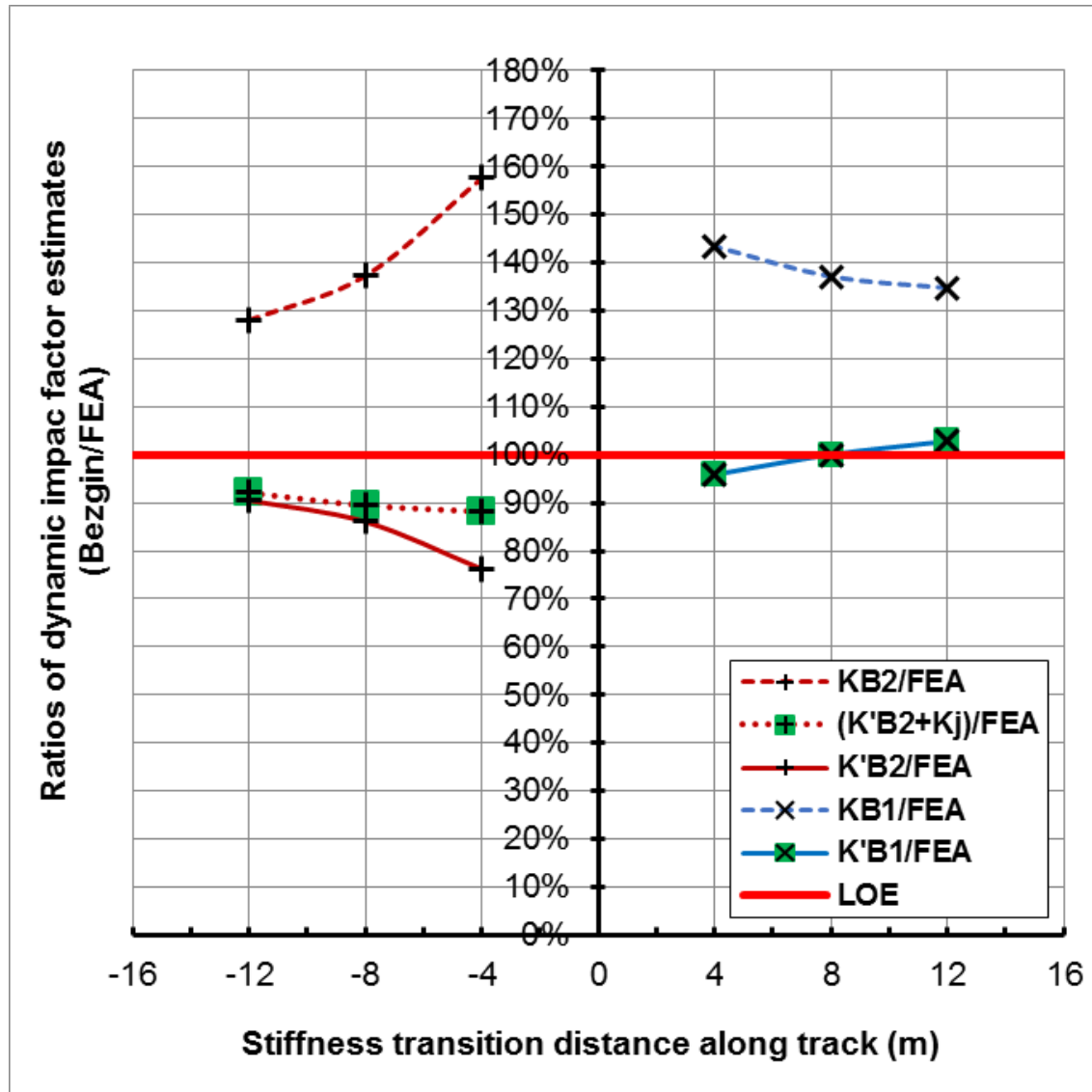
FINITE ELEMENT MODEL OF TRACK TRANSITION WITH PROFILE VARIATION AND ESTIMATED DYNAMIC IMPACT FORCE FACTORS
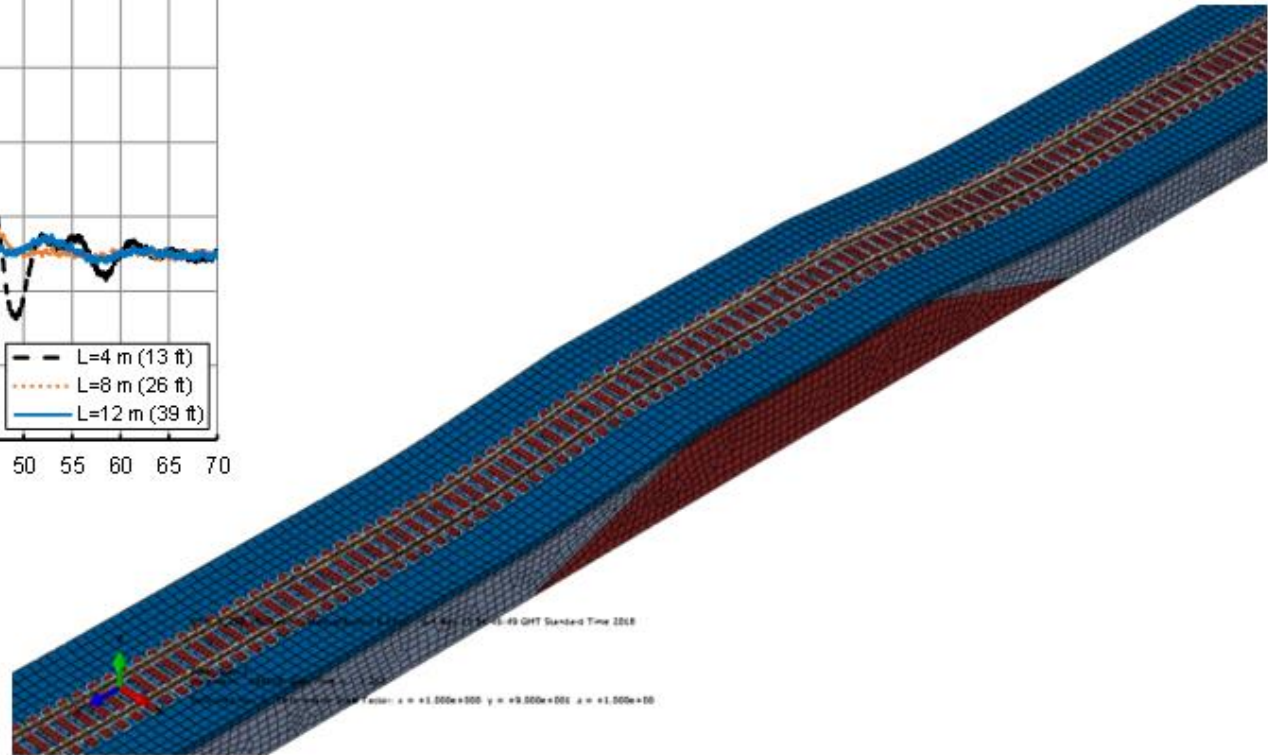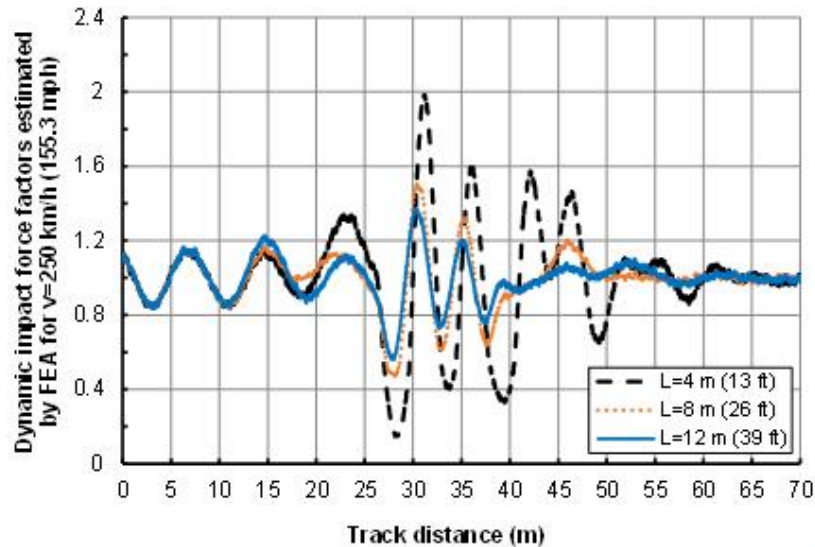
- The track profile ascends an amount **h=10.25 mm** (0.4 in) to the concrete structure along the entrance wedge and descends the same amount along the exit wedge at a speed of **v=250 km/h** (155.6 mph).

WHEEL FLATS

$$f = 1 - \frac{t_{fall}}{t_{pass}} = 1 - \frac{V}{L} \cdot \sqrt{\frac{2h}{g}}$$

$$f = 1 - \frac{t_{fall}}{t_{spin}} = 1 - \frac{4 \cdot v \cdot \sin\frac{\varphi}{2}}{\varphi \cdot \sqrt{r \cdot g}}$$
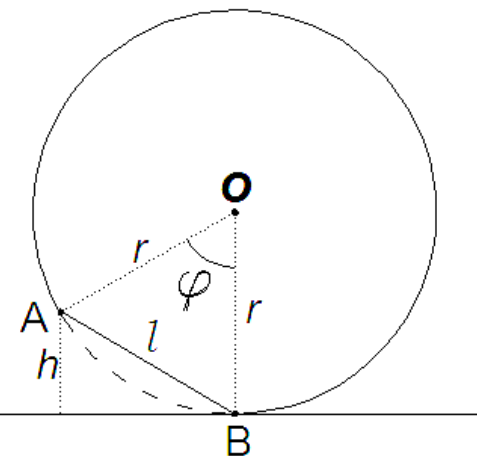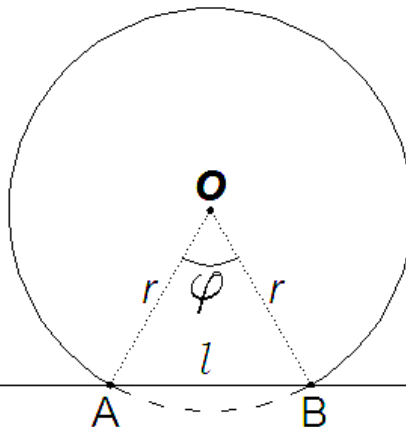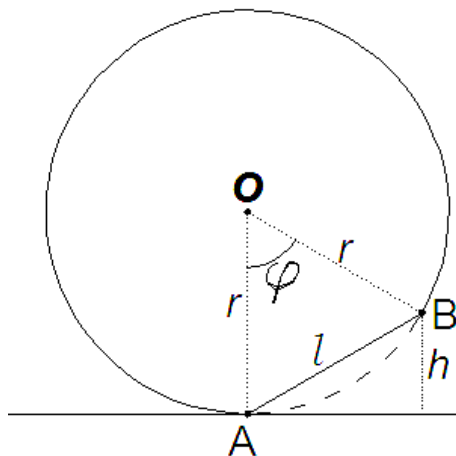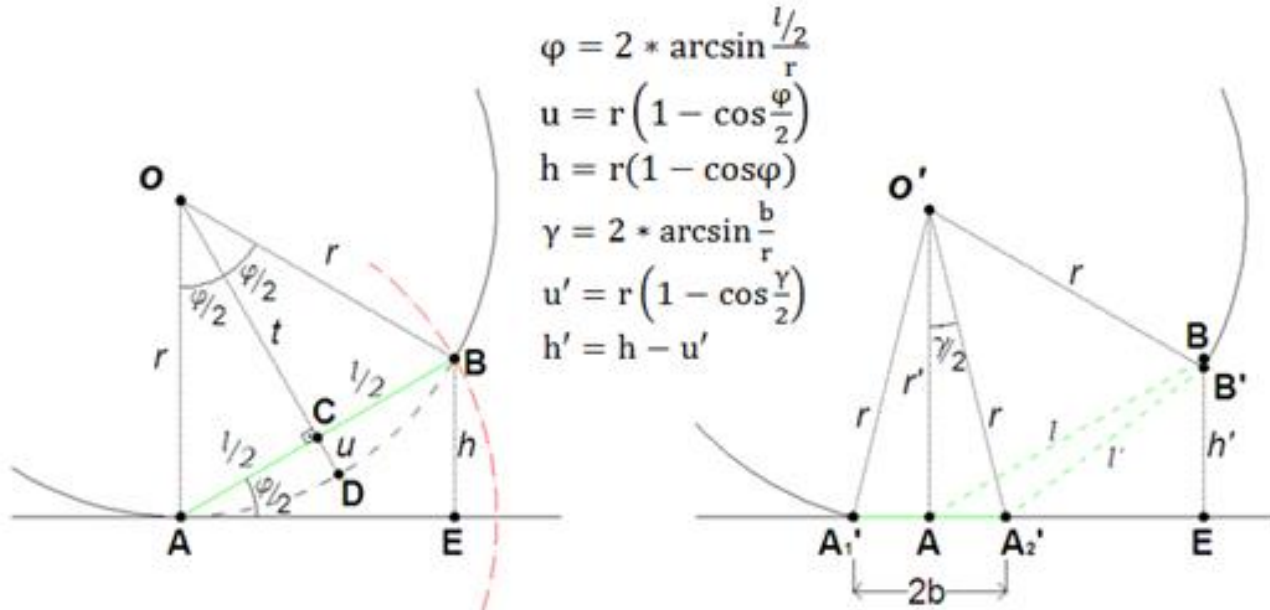
$$\varphi = 2 * \arcsin \frac{l/2}{r}$$

$$u = r\left(1 - \cos\frac{\varphi}{2}\right)$$

$$h = r(1 - \cos\varphi)$$

$$\gamma = 2 * \arcsin \frac{b}{r}$$

$$u' = r\left(1 - \cos\frac{\gamma}{2}\right)$$
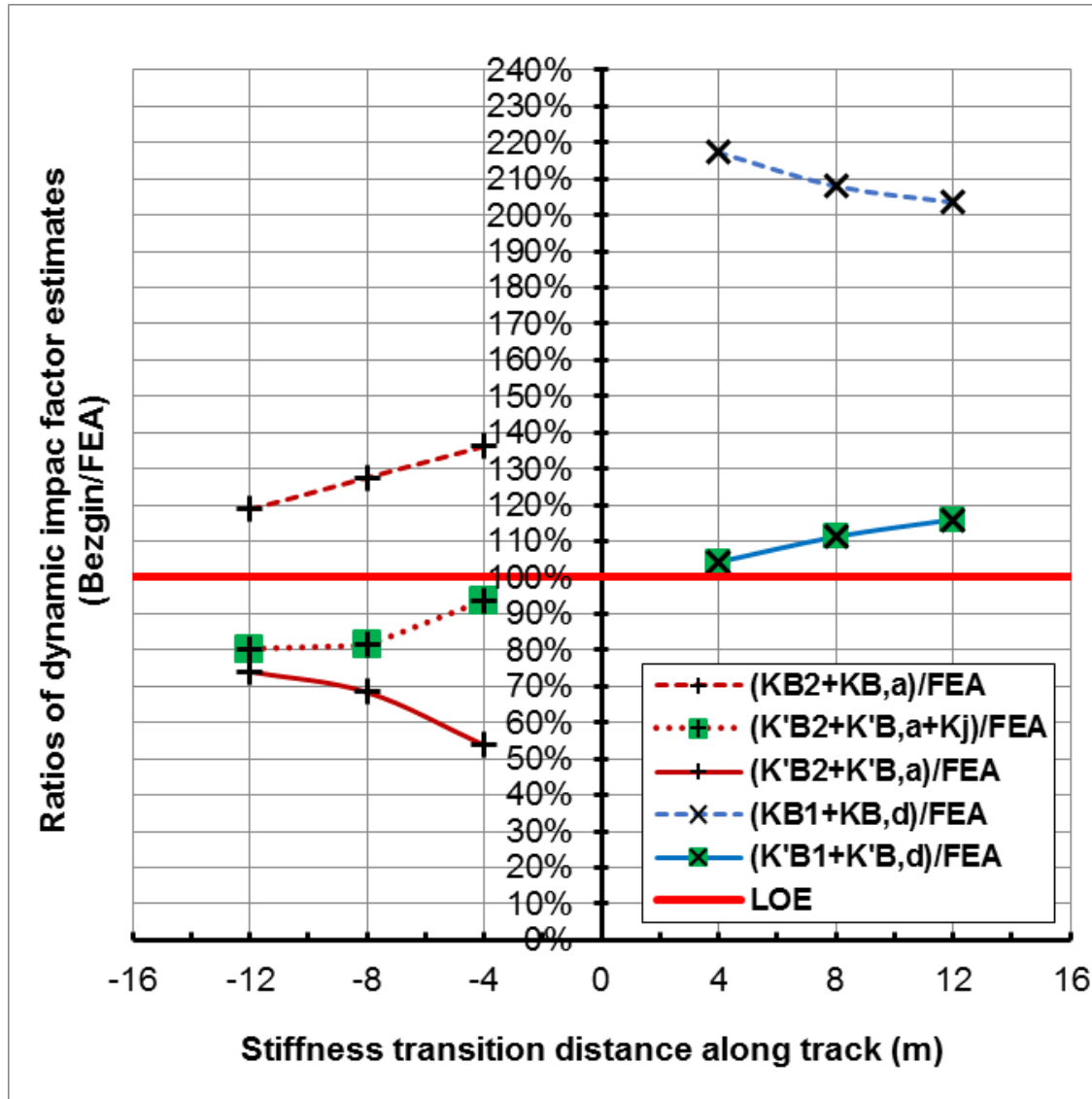
$$h' = h - u'$$

$$K'_{B3} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}}{a'.\varphi.\sqrt{g}}}$$

$$K'_{B3,H} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}\left(\frac{h'}{h}\right)}{a'.\varphi.\sqrt{g}}}$$

Manifestation of the calculation sequence:

K'_{B3} is estimated with **5 input parameters** to yield:

$F_{i1} = K'_{B3} * F_s = 2.05 * 80 \text{ kN} = \underline{164 \text{ kN}}$

I. $r = D/2 = 500 \text{ mm}$

II. $l = 90 \text{ mm}$

$$\varphi = 2 * arcsin \frac{l/2}{r} = 10.3^o = 0.18 \, rad$$

III. $v = 23 \text{ km/h}$

IV *and* V) $a'_1 = \dfrac{F_s}{k_{eq1}} = \dfrac{80 \text{ kN}}{0.95 \text{ kN/mm}} = 84.2 \text{ mm}$

$$K'_{B3} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}}{a'_1.\varphi.\sqrt{g}}} = 1 + 4.\sin\frac{10.3}{2}\sqrt{\frac{\sqrt{0.5}*\left(\frac{23}{3.6}\right)*\sin\frac{10.3}{2}}{0.0842*0.1801*\sqrt{9.81}}} = \mathbf{2.05}$$

5. First verification study

Uzzal, A. R., Ahmed, W., Rakheja, S. Dynamic Analysis of Railway Vehicle-Track Interactions due to Wheel Flat with a Pitch-Plane Vehicle Model. Journal of Mechanical Engineering. Vol. ME39, No.2, December 2008.

Zhai, W.M., Cai, C.B., Wang, Q., Lu, Z.W., Wu, X.S. Dynamic effects of vehicles on tracks in the case of raising train speed. Proceedings of the Institution of Mechanical Engineers, Part F, v 215, p.125-135, 2001.

- Authors of the first paper developed a two dimensional track model that employs the Rayleigh-Ritz method to analyze the vehicle-track system.
- Dynamic coupled system of the car supported on a bogie with suspension elements on an Euler-Bernoulli beam track on ballasted track.
- The wheel diameter is **D=840 mm** (33 in) and the train speed is **v=27 km/h** (16.8 mph). The static wheel force is **F_t=103.3 kN** (23.2 kips).

$$\frac{1}{k_{eq}} = \frac{1}{708} + \frac{1}{6.1} + \frac{1}{1377} + \frac{1}{100} \rightarrow k_{eq} = 5.7 \frac{kN}{mm} = 32.5 \frac{kip}{in} \qquad a = \frac{F_s}{k_{track}} = \frac{103.2\,kN}{100\frac{kN}{mm}} = 1\,mm = 0.04\,in$$

$$\varphi = 2 \cdot \arcsin\frac{l/2}{r} = 2 \cdot \arcsin\frac{52.6/2}{420} = 7.2\,\text{degrees}=0.126\,\text{radians} \qquad a' = \frac{F_s}{k_{eq}} = \frac{103.2\,kN}{5.7\,kN/mm} = 18.1\,mm = 0.7\,in$$

$$K_{B3} = 1 + 4.\sin\frac{\varphi}{2} \sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}}{a.\varphi.\sqrt{g}}} = 1 + 4 \cdot \sin\frac{7.2}{2} \sqrt{\frac{\sqrt{460} \cdot \frac{27 \cdot 1000}{3.6} \cdot \sin\frac{7.2}{2}}{1 + 0.126 \cdot \sqrt{9810}}} = 8.15$$

$$K'_{B3} = 1 + 4.\sin\frac{\varphi}{2} \sqrt{\frac{\sqrt{r}.v.\sin\frac{\varphi}{2}}{a'.\varphi.\sqrt{g}}} = 1 + 4 \cdot \sin\frac{7.2}{2} \sqrt{\frac{\sqrt{460} \cdot \frac{27 \cdot 1000}{3.6} \cdot \sin\frac{7.2}{2}}{18.1 \cdot 0.126 \cdot \sqrt{9810}}} = \boxed{2.68}$$

6. Second verification study
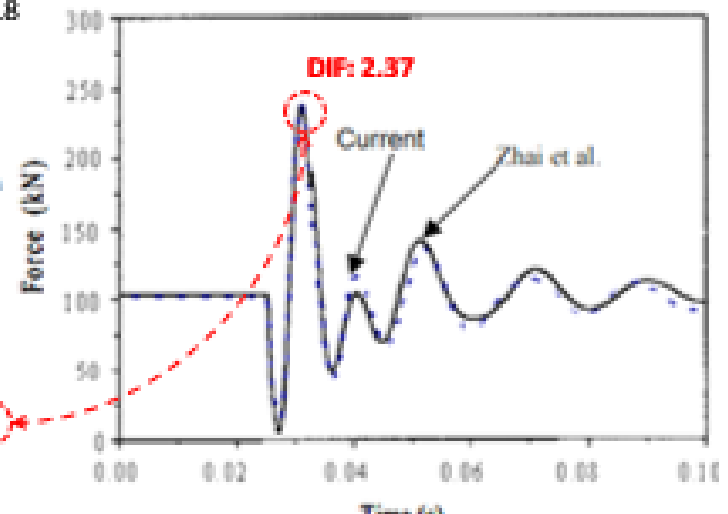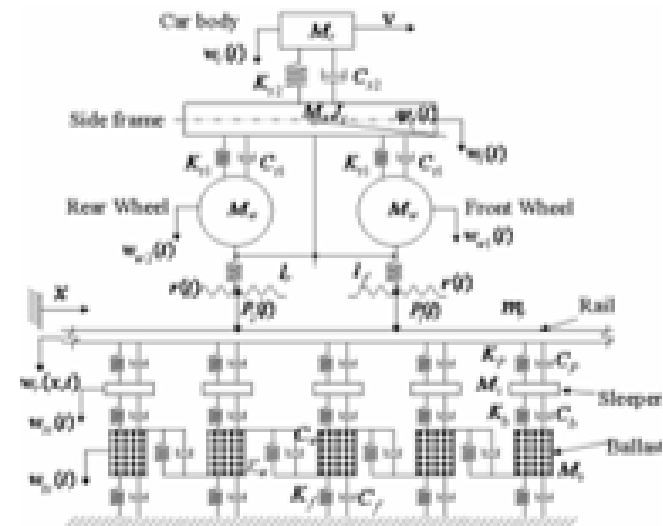
DIF: 2.37

Current

Zhai et al.

Force (kN)

6. Second verification study

- Bjan, J., Gu, Y., Murray, H.W. *A dynamic wheel-rail impact analysis of railway track under wheel flat by finite element analysis.* Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility. DOI: 10.1080/00423114.2013.774031. March 2013.
- Zilli, L., Zhao, X., Esveld, C., Dollevoet, R., Molodova, M. *Investigation into the causes of squats-correlation analysis and numerical modelling.* Wear. 265, p.1349-1355. 2008.
- Wheel spring stiffness used in this study is k_w=1.15 MN/m (6.6 kip/in), the rail pad stiffness is k_p=1300 MN/m (7,413 kip/in), the ballast stiffness per rail seat is k_b=45 MN/m (257 kip/in). This study excludes Hertzian contact stiffness.
- The rail is an Australian standard type AS-60 rail. The wheel diameter is 915 mm (36 in) and the center-to-center sleeper spacing is s=68.5 cm (2.25 ft). The authors conduct an array of analysis for train speed v=72 km/h (44.7 mph) and a static wheel force of F_s=128 kN (29 kips)
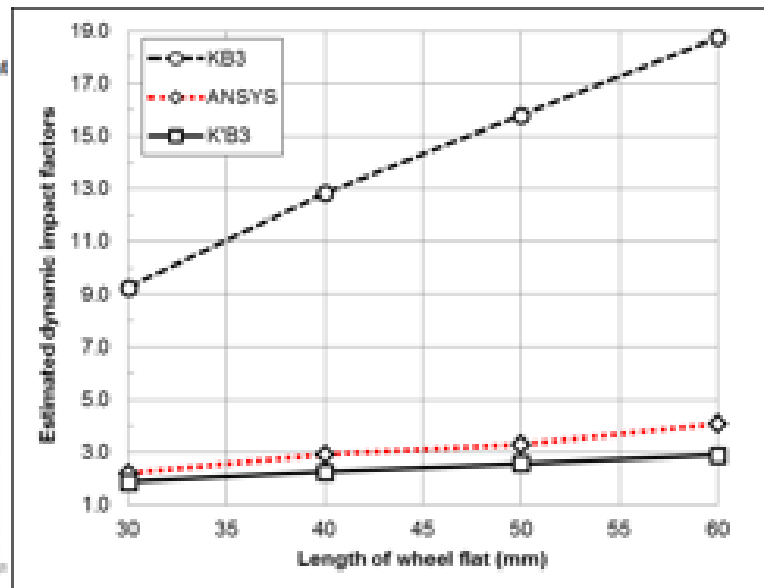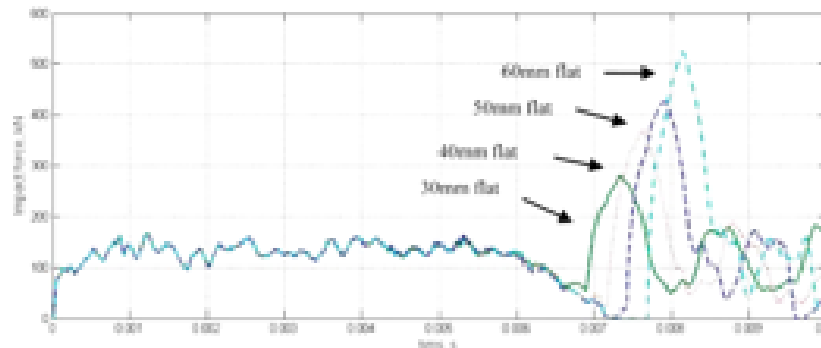
$$\frac{1}{k_{eq}} = \frac{1}{k_{track}} + \frac{1}{k_w} = \frac{1}{100} + \frac{1}{1.15} \rightarrow k_{eq} = 1.14 \frac{MN}{m} = 6.5 \; kip/in$$

$$a = \frac{F_s}{k} = \frac{127.9 \; kN}{100 \; N/mm} = 1.28 \; mm = 0.05 \; in$$

$$a' = \frac{F_s}{k_{eq}} = \frac{127.9 \; kN}{1.14 \; N/mm} = 112.2 mm = 4.4 \; in$$
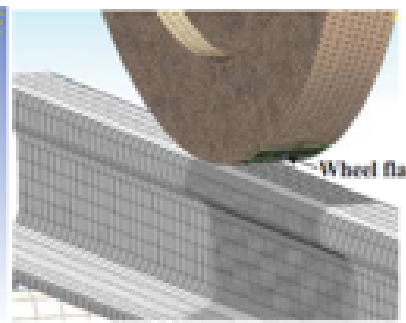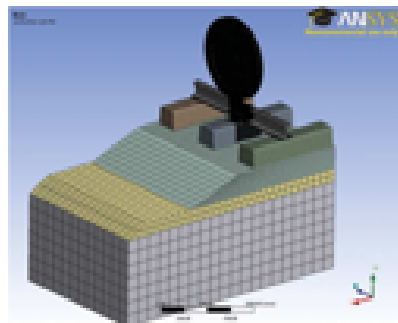
ESTIMATED DIF FOR WHEEL FLATS FOR D=920 mm and v=50 km/h
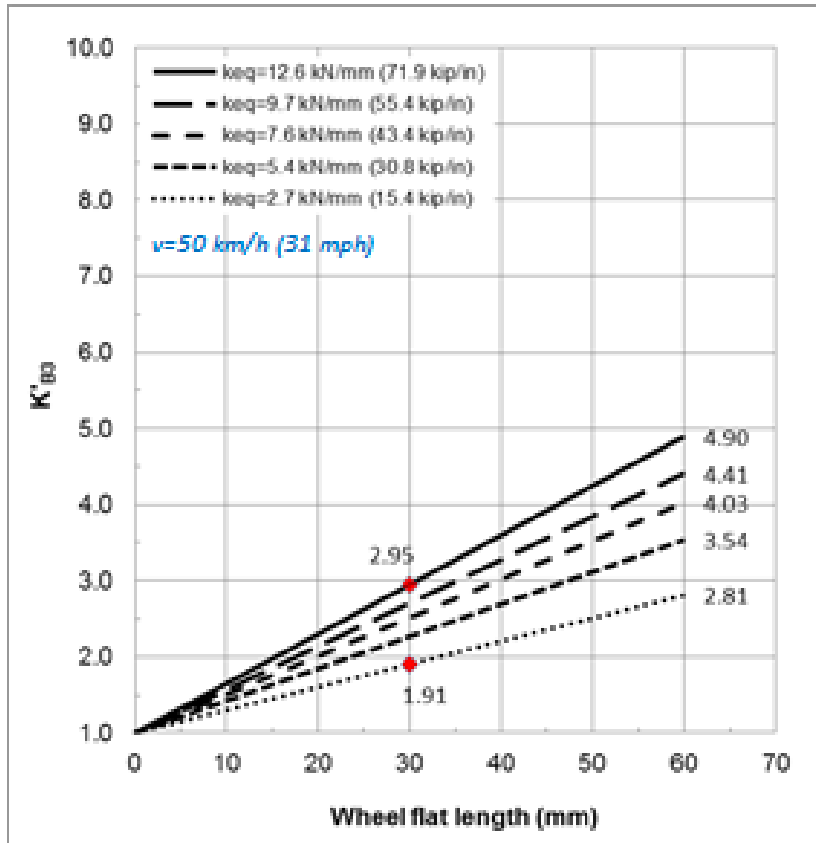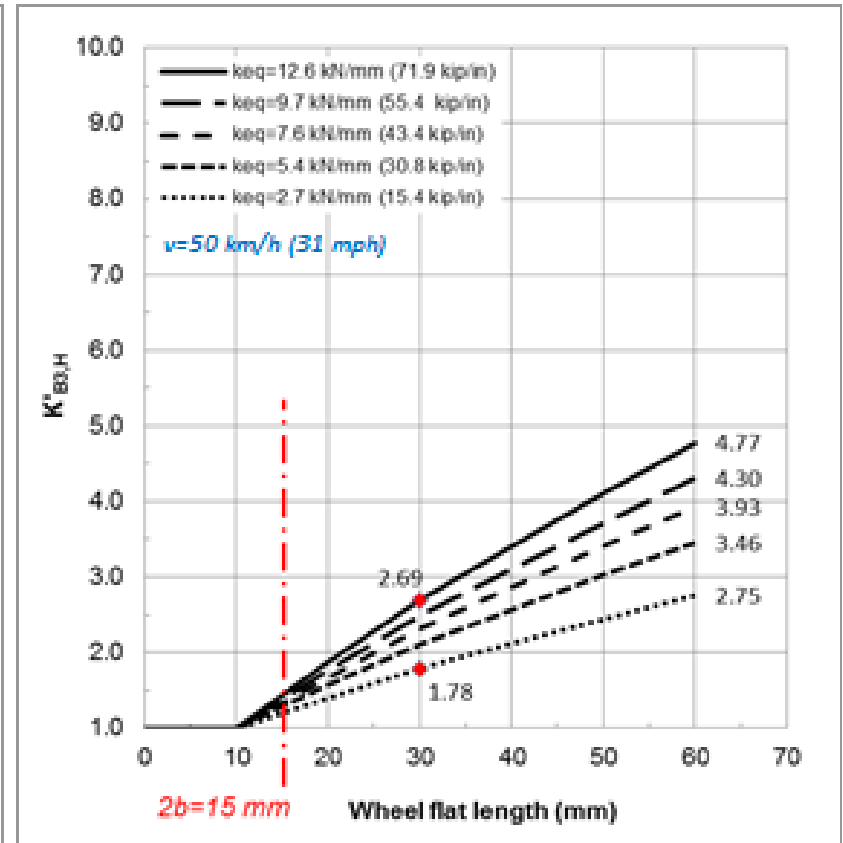
(a)   (b)

ESTIMATED DIF FOR WHEEL FLATS FOR D=920 mm and v=150 km/h
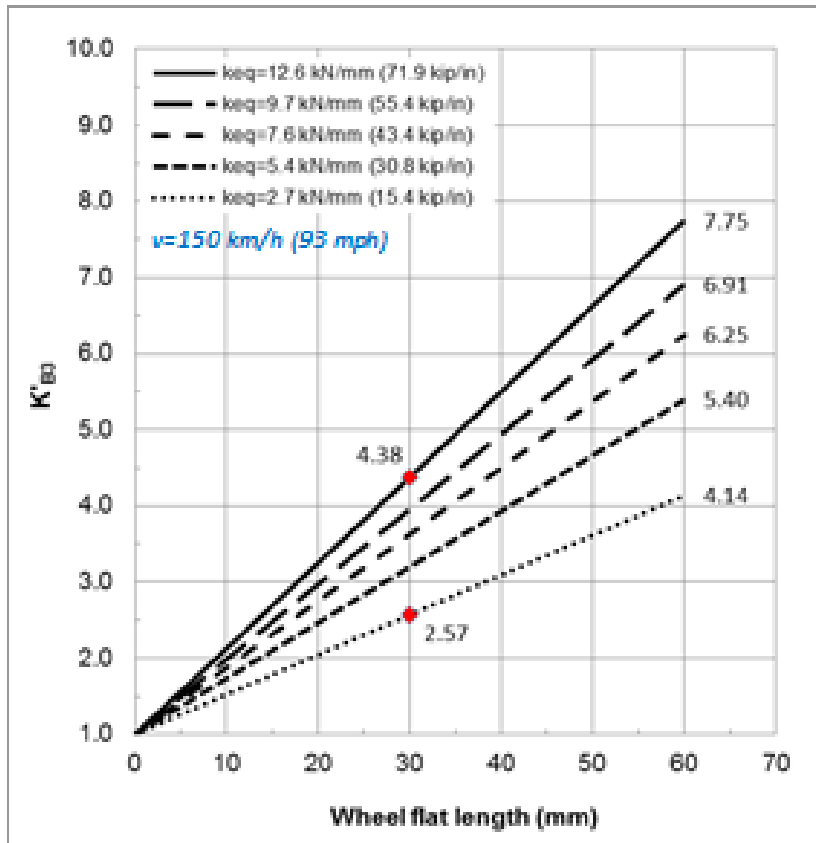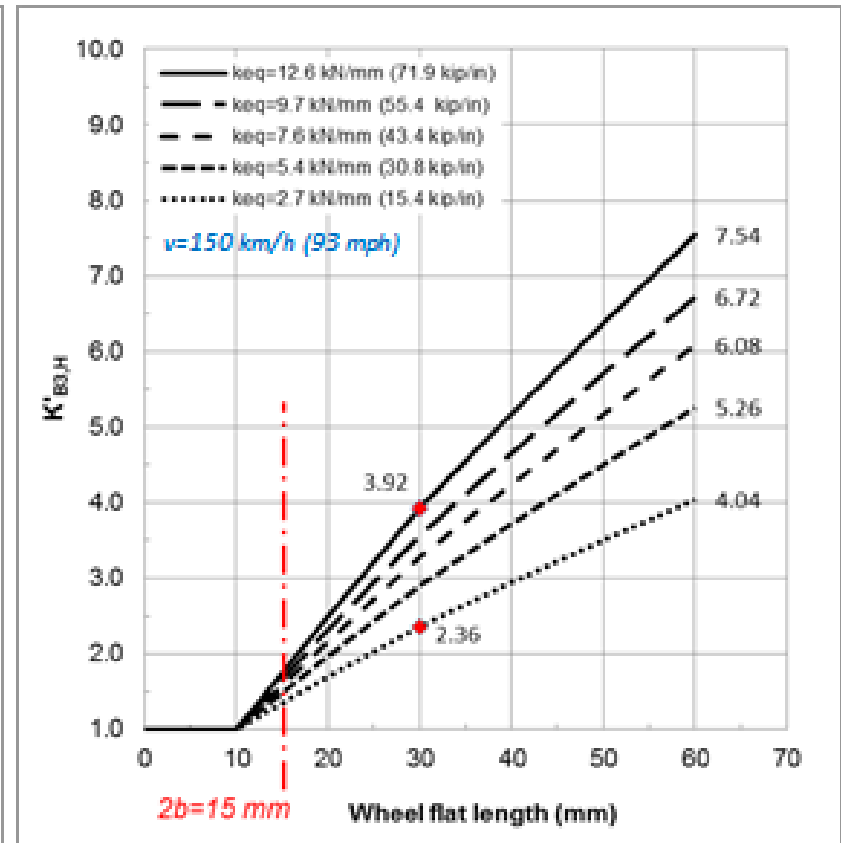
(c)   (d)

CONCLUSIONS

EXTENDED BEZGIN EQUATIONS FOR :
VARYING TRACK PROFILE AND TRACK STIFFNESS
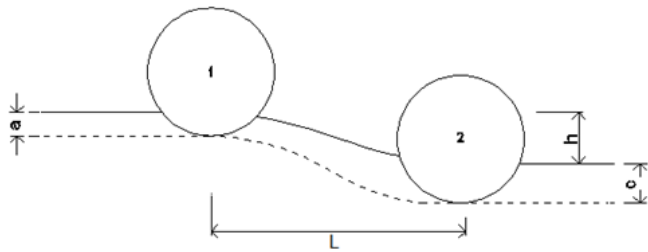
$$f = 1 - \frac{t_{fall}}{t_{pass}} = 1 - \frac{V}{L} \cdot \sqrt{\frac{2h}{g}}$$

$$K'_{B,d} = 1 + \sqrt{\frac{2h}{a'}(1 - f - s)}$$
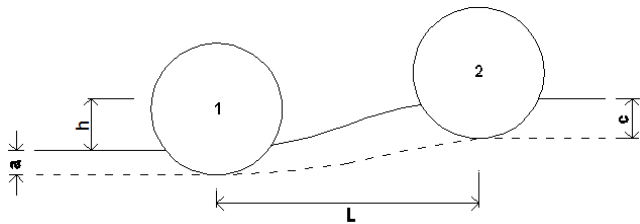
$$K'_{B,a} = 2\sqrt{\frac{h}{2a'}(1 - f - s) + 1} - 1$$

$$K_j = \frac{2h_a}{g} \cdot \left(\frac{v}{L}\right)^2$$
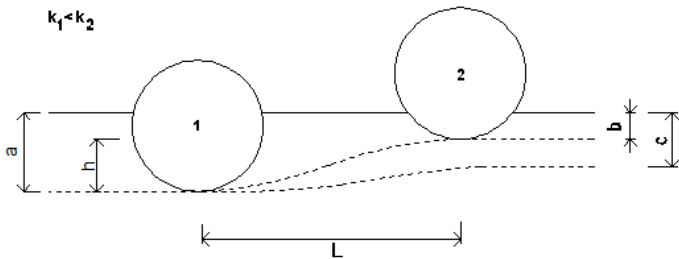
$$K'_{B2} = \sqrt{2\left[1 + f + s + \frac{a'}{b'} \cdot (1 - f - s)\right] - 1}$$
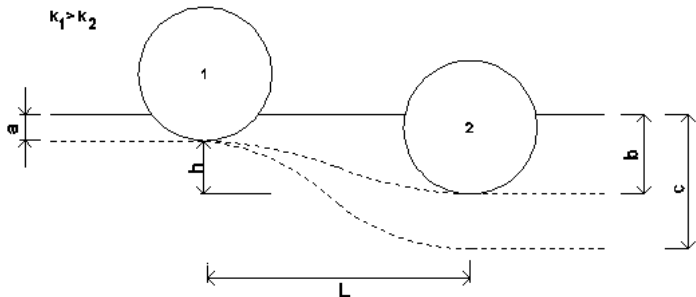
$$K'_{B1} = 1 + \sqrt{2\left[(1 - f - s)\left(1 - \frac{a'}{b'}\right)\right]}$$

BEZGIN – KOLUKIRIK EQUATIONS: K'$_{B3}$ and K'$_{B3,H}$
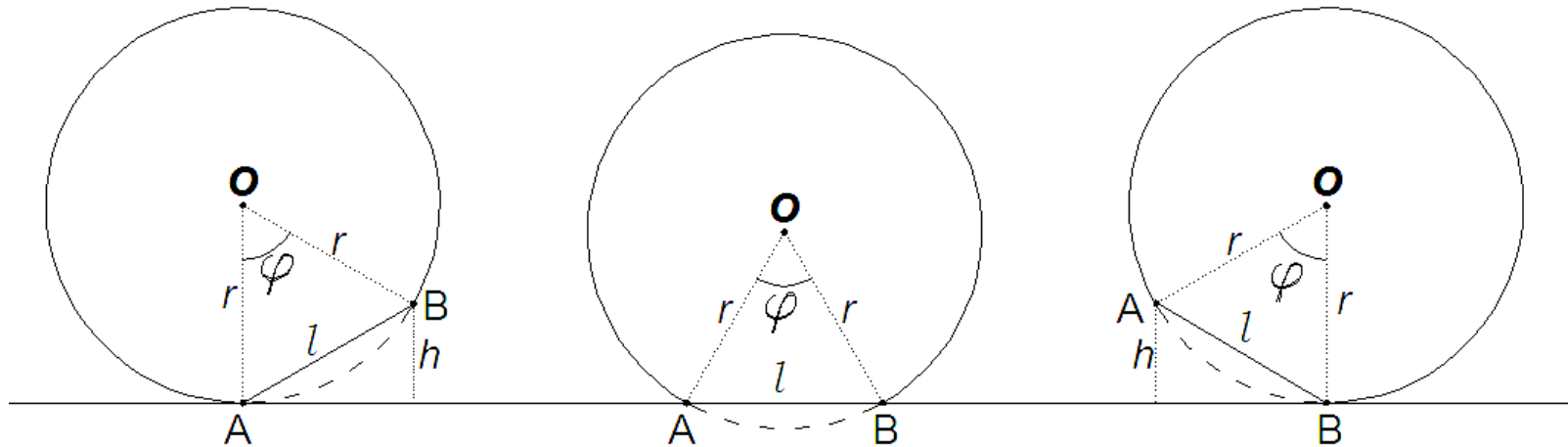
$$K'_{B3} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.\,v.\sin\frac{\varphi}{2}}{a'.\,\varphi.\sqrt{g}}}$$

$$K'_{B3,H} = 1 + 4.\sin\frac{\varphi}{2}\sqrt{\frac{\sqrt{r}.\,v.\sin\frac{\varphi}{2}\left(\frac{h'}{h}\right)}{a'.\,\varphi.\sqrt{g}}}$$

1. The proposed method by Bezgin, assists its beneficiary by providing the **ability** to **assess** the effects of track roughess on wheel forces and to **judge** the need to resort to more advanced methods.

2. it's **simplicity of application** and **economic requirements for its presence** in relation to its **capabilities** provides a useful asset for its beneficiary.

3. The developed equations show that **equivalent stiffness** of bogie-wheel-rail system is **extremely effective** in limiting the developing impact forces. This highlights the importance of **proper selection and functioning** of wheel and bogie stiffness elements.

4. Study also showed the profound effect of the **secondary dynamic impact forces** that develop due to **track ascent** especially for transition lengths: **L ≤ 8m** *(26 ft)* (14 sleepers spaced at 2 ft o.c), which are prone to generate significant dynamic impact forces.

5. **Less than a minute** needed to manually estimate dynamic impact forces.

6. A small number of highly deterministic and reasonably estimable parameters required:

    I. *Train speed (v)*

    II. *Stiffness (track, primary, secondary, Hertzian) (k, k_w , k_b , k_H)*

    III. *Static axle force (F_s)*

    IV. *Track roughness values (h and L)*

    V. *Wheel diameter*

    VI. *Length of wheel flat*

7. The proposed method and the resultant equations effectively capture and mathematically represent the effect of system stiffness (railway track and rolling stock) on the dynamic impact forces.

8. The resultant equations also represent the relative effect of a given roughness with respect to system stiffness. in other words, **the stiffer is the system, the lower is a' and therefore the higher is K'$_{B,d}$.**
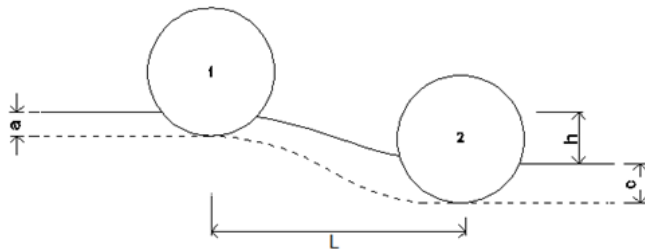
$$K'_{B,d} = 1 + \sqrt{\frac{2h}{a'}(1 - f - s)}$$

9. As a corollary, the equations also capture the effect of static wheel force on the dynamic impact force factor. in other words, **the higher is the static axle force, the higher is a' and therefore the lower is the dynamic impact force factor.**

Transportation Geotechnics and Geoecology, TGG 2017, 17-19 May 2017, Saint Petersburg, Russia

Development of a new and an explicit analytical equation that estimates the vertical impact loads of a moving train

Dr. Niyazi Özgür Bezgin*

Istanbul University, Civil Engineering Department, Avcılar Campus, 34320, Istanbul

Abstract

One can only estimate the dynamic vertical impact loads under motion, since there are many effective parameters some of which are unrepresented in an equation and since the values of the considered parameters are not deterministic but estimations. Many empirical and semi-empirical equations in the literature correlate dynamic impact loads to train speed and measurable aspects of train and track components. These aspects frequently relate to track and train geometry and stiffness. However, the development of these equations relies on load and deflection measurements from particular in-service tracks or especially set-up test tracks. The constants that frequently appear in these equations are particular to the conditions that generated them. Therefore, one lacks an explicit understanding of these equations unless one takes the time to investigate in detail the particular study and the particular set of data that generated these equations. Train speed limits also bound the applicability of these equations. This paper

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Abstract

Effects of Varying Track Stiffness on

Advancement and Application of the Bezgin Method to Estimate Effects of Stiffness Variations along Railways on Wheel Forces

Niyazi Özgür Bezgin, Mohamed Wehbi

First Published April 26, 2019 | Research Article | 🔄 Check for updates
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Abstract

The need for an analytical method that one can apply manually to estimate dynamic impact forces on railway tracks that occur because of varying track stiffness or track profile initiated a study to develop an analytical method named as the Bezgin Method. The advancement of this method presented in this paper includes an extension of a set of equations developed and introduced by the first author earlier as the Bezgin Equations using the proposed method and development of a new equation. In addition to track stiffness taken into

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Abstract

Proposal and Application of a New Technique to Forecast Railway Track Damage Because of Track Profile Variations

Mohamed Wehbi, Niyazi Özgür Bezgin

Article information ⌄

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Abstract

This paper presents a new technique to estimate dynamic impact forces on railway tracks that develop because of variations in track profile. The approach presented uses a wavelet decomposition method to systematically define the irregular profile variation of a rough track length in relation to regular wavelet functions. These functions provide the regular profile variation parameters to estimate the dynamic impact

1 **DEVELOPMENT OF THE BEZGIN-KOLUKIRIK EQUATION TO ESTIMATE DYNAMIC**
2 **IMPACT FORCES GENERATED BY WHEEL FLATS ON RAILWAY TRACKS**
3

4 **Niyazi Özgür Bezgin, Ph.D (Corresponding author)**
5 Associate Professor
6 İstanbul University-Cerrahpaşa, Avcilar Campus
7 Department of Civil Engineering, 34320, Avcilar, Istanbul, Turkey
8 Tel: 00.90.533.663.9755; E-mail: ozgur.bezgin@istanbul.edu.tr
9

10 **Cengiz Kolukırık, M.Sc**
11 Ph.D Student
12 İstanbul Gelişim University
13 Department of Civil Engineering, 34310, Avcilar, Istanbul, Turkey
14 E-mail: cengizkolukirik@gmail.com
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16

17 **98th Transportation Research Board Meeting**
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19 **Railroad Track Maintenance Research**
20 **Standing Committee on Railway Maintenance (AR060)**
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22 **Paper: 19-01728**
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25 Word count: 7247 words text (abstract + text + references + table titles) + 1 table * 250 words (each) =
26 7497 words

RAPID ANALYTICAL ASSESSMENT OF BEARING STRESSES ON RAILWAY TRACK COMPONENTS

ASSESSMENT OF BEARING PRESSURES

1. What is the average and the maximum values of the bearing pressure under the sleeper on the ballast?

2. What is the contact pressure between the ballast particles?

3. What is the transferred pressure to the subgrade / natural soil?
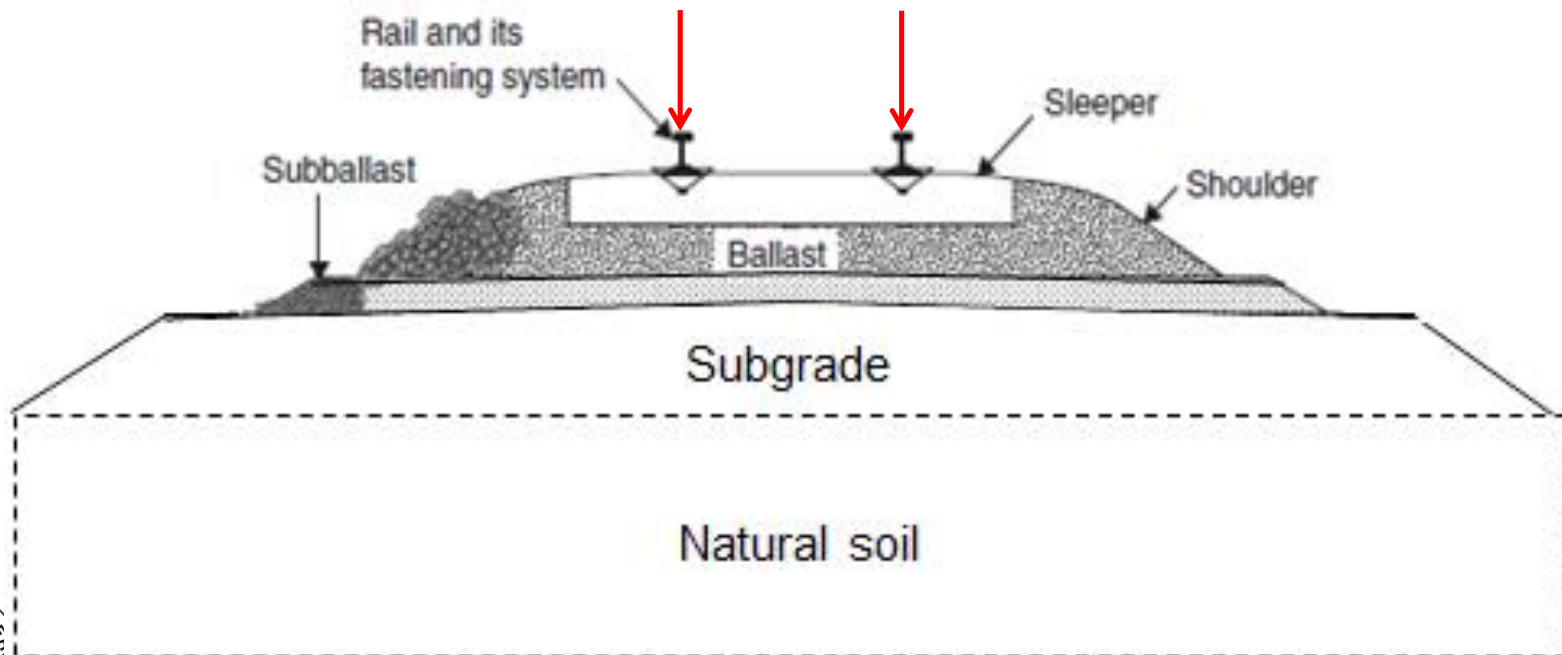
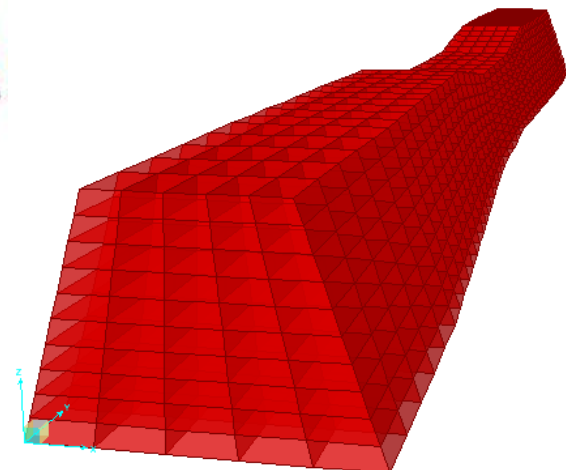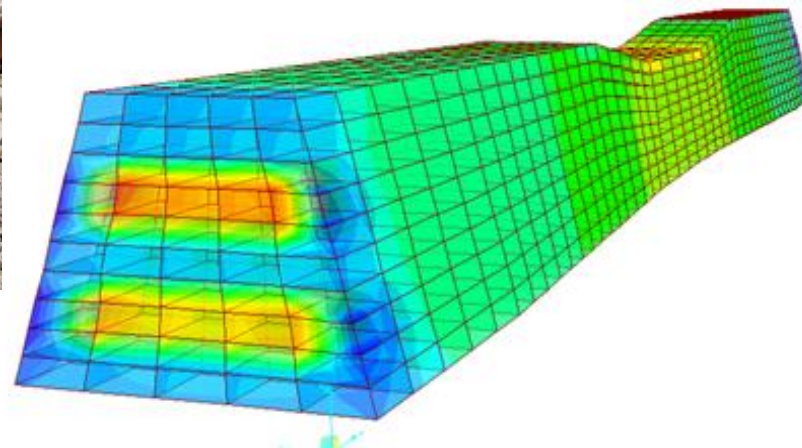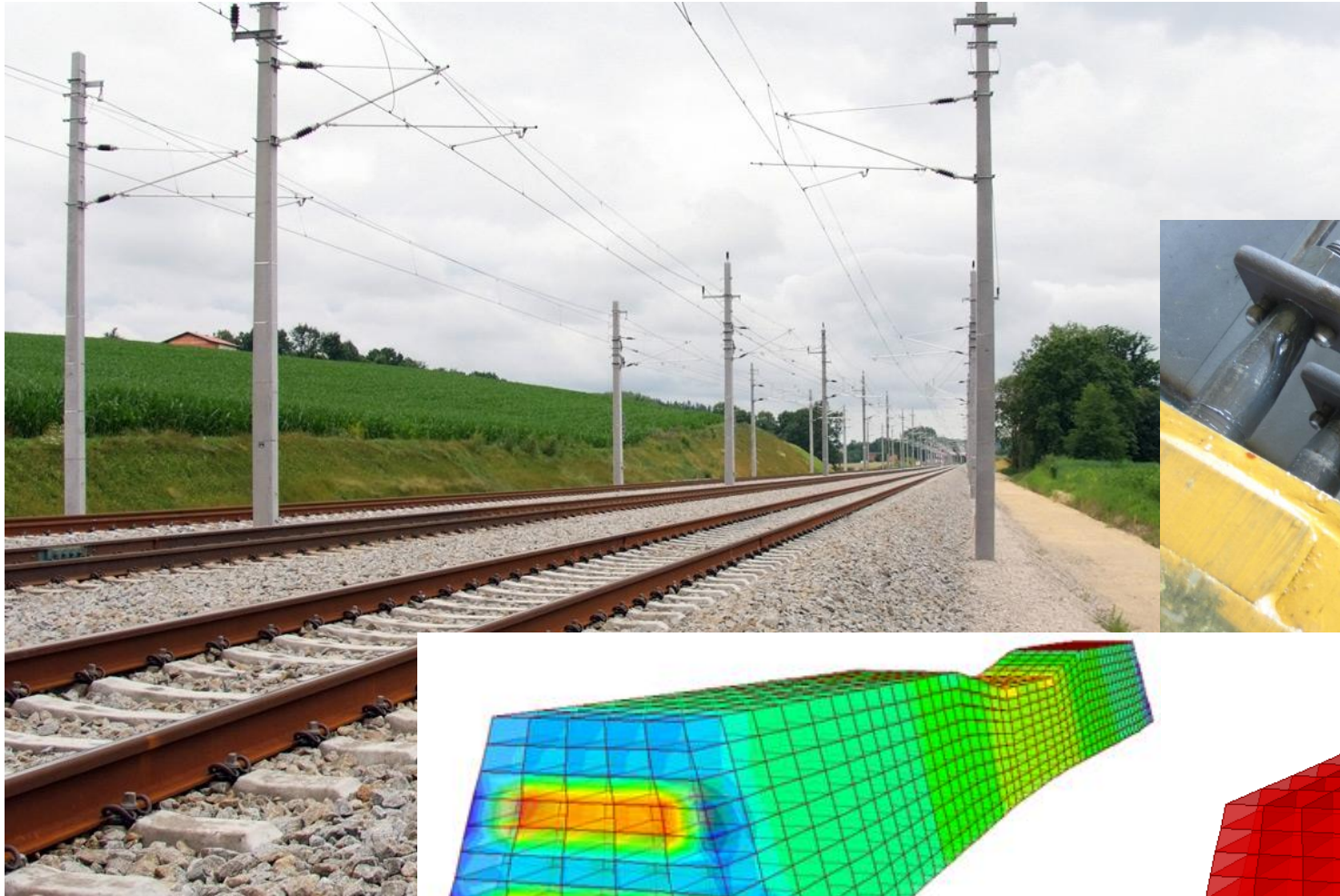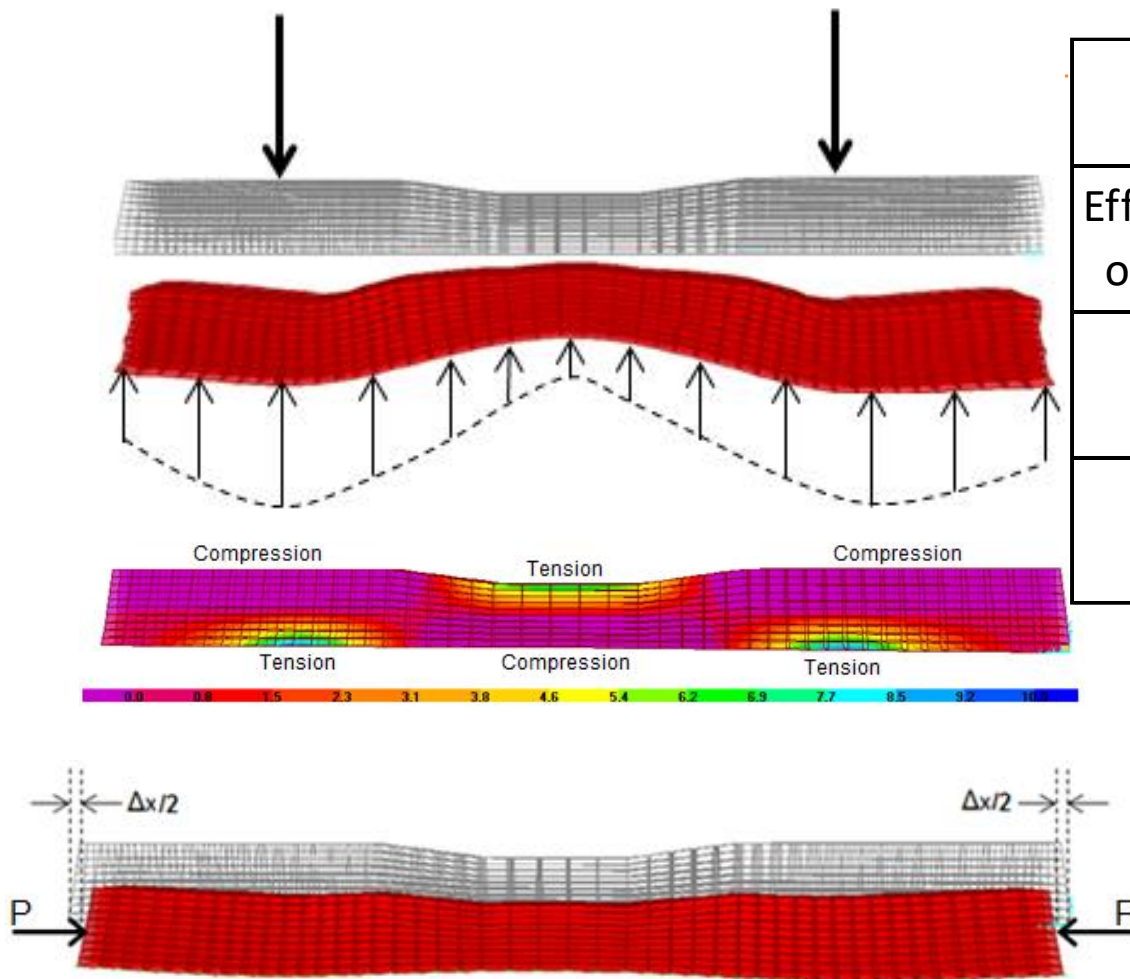| Static axle force = | 22.5 | Ton-f |
	49.6	Kips
Effective static axle force on the sleeper below =	11.3	Ton-f
	24.8	Kips
Sleeper base area =	0.70	m^2
	7.53	ft^2
Effective base area =	0.53	m^2
	5.65	ft^2

- Effective base area **~ 75%** of geometric base area

- Let us say that **50%** goes to the sleeper under the wheels.

AXLE FORCE VARYING WITH THE ESTIMATED DYNAMIC IMPACT FORCE FACTOR (DIF) BY THE BEZGIN METHOD

DIF	Axle force		Sleeper bearing stresses				Ballast particle contact stresses	
			Average		Maximum			
	Ton-f	Kips	Ton-f/m^2	psi	Ton-f/m^2	psi	Ton-f/m^2	psi
1	11.3	24.8	16.1	22.9	21.4	30.5	107.1	152.4
1.5	16.9	37.2	24.1	34.3	32.1	45.7	160.7	228.6
2	22.5	49.6	32.1	45.7	42.9	61.0	214.3	304.8
2.5	28.1	62.0	40.2	57.1	53.6	76.2	267.9	381.0
3	33.8	74.4	48.2	68.6	64.3	91.4	321.4	457.2
3.5	39.4	86.8	56.3	80.0	75.0	106.7	375.0	533.4

Subgrade bearing stresses			
Average		Maximum	
Ton-f/m^2	psi	Ton-f/m^2	psi
5.4	7.6	7.1	10.1
8.0	11.4	10.7	15.2
10.7	15.2	14.3	20.3
13.4	19.0	17.8	25.4
16.1	22.8	21.4	30.4
18.7	26.6	25.0	35.5

- *Through the use of a preferred method: (Talbot, Boussinesq, Tschebotarioff, FEM, DEM) ballast and subballast thickness is chosen such that the tie pressure reduces to 1/3rd the value under the tie.*

- *Assuming that only 20% of the tie area is in actual contact with the ballast*

ASSESSMENT OF WHEEL-RAIL CONTACT INTERFACE BEARING STRESSES

- One can estimate the contact area from **Hertz Theorem**

- Typical wheel-rail interface bearing area **~ 2 cm² ~** *0.3 in²*

- Rail steel strength **~ 900 MPa – 1,400 MPa** *(130,000 psi – 203,000 psi)*

DIF by K'$_{B3}$	Wheel force		Hertz contact stress	
	Ton-f	*Kips*	*MPa*	*ksi*
1.0	11.3	24.8	741	143
1.5	16.9	37.2	906	175
2.0	22.5	49.6	1,045	202
2.5	28.1	62.0	1,168	225
3.0	33.8	74.4	1,281	247
3.5	39.4	86.8	1,383	267

- *Initiation of rail damage, rail battering, plastification and likely plastic flow.*

- Hertz contact stresses estimated through Hertzwin® 2.6.4

- Wheel diameter = 920 mm

ALLTRACK® V1

ALLTRACK® V1

- A workbook developed to estimate dynamic impact forces on railway tracks due to track and wheel roughness.

- **ALLTRACK® V1** can be downloaded at:

https://www.researchgate.net/publication/330083115_ALLTRACK_V1_January_2_2019

Teşekkür ederim   Thank you

ozgur.bezgin@istanbul.edu.tr